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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 7

# Coordinate Geometry

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

## INSIDE THIS BOOKLET


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## HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check a found point by putting it back: equal distances must come out equal, and a point that divides $AB$ internally must lie between $A$ and $B$.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

# Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

### HOW MARKS ARE GIVEN IN COORDINATE GEOMETRY ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Distance (2 marks):** formula with the values put in 1 · answer (both values if there are two) 1.
3. **Equidistant point (2–3 marks):** $PA^2 = PB^2$ written out 1 · simplified equation 1 · the point 1.
4. **Ratio by the section formula (3 marks):** point in terms of $k$ 1 · $k$ and the ratio 1 · the point 1.
5. **Type of figure (3–4 marks):** all sides 1–2 · diagonals 1 · conclusion with the reason 1.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

# A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

## Distance formula

| Q | Answer | Working / reason |
|---|---|---|
| A1 | **(c)** 10 | $\sqrt{36+64} = \sqrt{100} = 10$. |

**A2.** Find the distance between the points: **(i)** $(3,\ 4)$ and $(-5,\ -2)$ **(ii)** $(a,\ b)$ and $(-a,\ -b)$ [2]

(i) $\sqrt{(-5-3)^2 + (-2-4)^2} = \sqrt{64+36} = 10$ units — 1

(ii) $\sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{4a^2+4b^2} = 2\sqrt{a^2+b^2}$ — 1

**A3.** Check whether the points $A(1,\ 5)$, $B(2,\ 3)$ and $C(-2,\ 11)$ are collinear. [2]

$AB = \sqrt{1+4} = \sqrt{5}$, $AC = \sqrt{9+36} = 3\sqrt{5}$, $BC = \sqrt{16+64} = 4\sqrt{5}$ — 1

$AB + AC = 4\sqrt{5} = BC$, so the points are collinear — 1

**A4.** Show that $A(2,\ 1)$, $B(5,\ 3)$, $C(3,\ 6)$ and $D(0,\ 4)$ are the vertices of a square. [3]

$AB = \sqrt{9+4} = \sqrt{13}$, $BC = \sqrt{4+9} = \sqrt{13}$ — ½

$CD = \sqrt{9+4} = \sqrt{13}$, $DA = \sqrt{4+9} = \sqrt{13}$ — ½

Diagonals: $AC = \sqrt{1+25} = \sqrt{26}$, $BD = \sqrt{25+1} = \sqrt{26}$ — 1

All four sides equal and the diagonals equal: $ABCD$ is a square — 1

**A5.** Find the values of $x$ for which the distance between $P(x,\ 3)$ and $Q(4,\ -1)$ is 5 units. [2]

$(x-4)^2 + 16 = 25 \rightarrow (x-4)^2 = 9$ — 1

$x - 4 = \pm 3 \rightarrow x = 7$ or $x = 1$ — 1

**A6.** Find the point on the $y$-axis which is equidistant from $A(5,\ -2)$ and $B(-3,\ 2)$. **[3]**

Let $P(0,\ y)$: $PA^2 = PB^2 \rightarrow 25 + (y+2)^2 = 9 + (y-2)^2$ — 1

$25 + 4y + 4 = 9 - 4y + 4 \rightarrow 8y = -16 \rightarrow y = -2$ — 1

The point is $(0,\ -2)$ (check: $PA = PB = 5$) — 1

**A7.** Find a relation between $x$ and $y$ such that the point $(x,\ y)$ is equidistant from $(1,\ 3)$ and $(5,\ 1)$. **[2]**

$(x-1)^2 + (y-3)^2 = (x-5)^2 + (y-1)^2 \rightarrow -2x - 6y + 10 = -10x - 2y + 26$ — 1

$8x - 4y = 16$, i.e. $2x - y = 4$ — 1

**A8.** Name the type of quadrilateral formed by $A(-2,\ 0)$, $B(2,\ 3)$, $C(6,\ 0)$ and $D(2,\ -3)$. Give reasons. **[3]**

$AB = \sqrt{16+9} = 5$, $BC = \sqrt{16+9} = 5$, $CD = 5$, $DA = 5$ — 1

Diagonals: $AC = 8$, $BD = 6$ — 1

Four equal sides but unequal diagonals: a **rhombus** — 1

**A9.** Show that $A(3,\ 0)$, $B(6,\ 4)$ and $C(-1,\ 3)$ are the vertices of a right-angled isosceles triangle. **[3]**

$AB = \sqrt{9+16} = 5$, $AC = \sqrt{16+9} = 5$, $BC = \sqrt{49+1} = \sqrt{50}$ — 1

$AB = AC$: isosceles — 1

$AB^2 + AC^2 = 50 = BC^2$: right-angled at $A$ — 1

## Section formula

| Q | Answer | Working / reason |
|---|---|---|
| A10 | **(b)** $(2,\ 2)$ | $\left(\frac{-2+6}{2}, \frac{7-3}{2}\right) = (2,\ 2)$. |

**A11.** Find the coordinates of the point which divides the join of $(-2,\ 3)$ and $(8,\ -7)$ in the ratio $3 : 2$. **[2]**

$x = \frac{3(8) + 2(-2)}{5} = 4$ — 1

$y = \frac{3(-7) + 2(3)}{5} = -3$; the point is $(4,\ -3)$ — 1

**A12.** Find the points of trisection of the segment joining $A(-3,\ 4)$ and $B(6,\ -2)$. **[3]**

$P$ divides $AB$ in $1 : 2$: $P\left(\frac{6-6}{3}, \frac{-2+8}{3}\right)$ — 1

$P = (0,\ 2)$ — ½

$Q$ divides $AB$ in $2 : 1$: $Q\left(\frac{12-3}{3}, \frac{-4+4}{3}\right)$ — 1

$Q = (3,\ 0)$ — ½

**A13.** In what ratio does the point $P(2,\ 1)$ divide the join of $A(-4,\ 5)$ and $B(5,\ -1)$? **[3]**

Let the ratio be $k:1$: $x = \dfrac{5k-4}{k+1}$ — 1

$\dfrac{5k-4}{k+1} = 2 \to 3k = 6 \to k = 2$: the ratio is $2:1$ — 1

Check: $y = \dfrac{-2+5}{3} = 1$ ✓ — 1

**A14.** Find the ratio in which the $y$-axis divides the join of $(6,\ -2)$ and $(-3,\ 7)$. Also find the point of division. **[3]**

Ratio $k:1$: $x = \dfrac{-3k+6}{k+1}$ — 1

$x = 0 \to k = 2$: the ratio is $2:1$ — 1

$y = \dfrac{2(7)+1(-2)}{3} = 4$: the point is $(0,\ 4)$ — 1

**A15.** The points $A(2,\ 3)$, $B(5,\ 4)$, $C(x,\ 6)$ and $D(1,\ y)$, taken in order, are the vertices of a parallelogram. Find $x$ and $y$. **[2]**

Diagonals bisect each other: $\left(\dfrac{2+x}{2},\ \dfrac{9}{2}\right) = \left(\dfrac{6}{2},\ \dfrac{4+y}{2}\right)$ — 1

$x = 4,\ y = 5$ — 1

**A16.** $A(-1,\ 2)$ and $B(5,\ -6)$ are the ends of a diameter of a circle. Find its centre and radius. **[2]**

Centre = midpoint of $AB = (2,\ -2)$ — 1

Radius $= \sqrt{(5-2)^2 + (-6+2)^2} = 5$ units — 1

**A17.** Find the coordinates of the points which divide the join of $A(-4,\ 0)$ and $B(0,\ 8)$ into four equal parts. **[3]**

Midpoint $Q$ of $AB$: $(-2,\ 4)$ — 1

Midpoint of $AQ$: $(-3,\ 2)$ — 1

Midpoint of $QB$: $(-1,\ 6)$. The points are $(-3,\ 2)$, $(-2,\ 4)$, $(-1,\ 6)$ — 1

B

PART B · SOLUTIONS

# Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

## Multiple choice

| Q | Answer | Working / reason |
|---|---|---|
| B1 | **(c)** an isosceles triangle | Sides 6, 5, 5: two equal, and $25 + 25 \neq 36$, so isosceles but not right-angled. |
| B2 | **(c)** third quadrant | $\left(\frac{-8+2}{3}, \frac{2-5}{3}\right) = (-2,\ -1)$: Quadrant III. |
| B3 | **(a)** $\pm 4$ | $9 + p^2 = 25 \rightarrow p = \pm 4$. |
| B4 | **(b)** $(0,\ 12)$ | $4 + (y-3)^2 = 36 + (y-5)^2 \rightarrow 4y = 48 \rightarrow y = 12$. |
| B5 | **(d)** $-4$ | Midpoint $= (-2,\ 3)$, so $\frac{a}{2} = -2 \rightarrow a = -4$. |
| B6 | **(a)** $(0,\ 5)$ | $D = A + C - B = (-2+5-3,\ 1+6-2) = (0,\ 5)$. |
| B7 | **(d)** $2 : 3$ | $\frac{6k-4}{k+1} = 0 \rightarrow k = \frac{2}{3}$, so $2 : 3$. |
| B8 | **(a)** | Both true; R is the reason for A. |

## Short answer with reasons

**B9.** Is the point $P(2,\ -1)$ equidistant from $A(-1,\ 3)$ and $B(6,\ 2)$? Justify. **[2]**

$PA = \sqrt{9+16} = 5,\ PB = \sqrt{16+9} = 5$ — 1

$PA = PB$, so yes, $P$ is equidistant — 1

**B10.** Find the point $Q$ on the $x$-axis which lies on the perpendicular bisector of the segment joining $A(-3,\ 4)$ and $B(5,\ 4)$. What type of triangle is $ABQ$? **[2]**

$Q(x,\ 0)$ with $QA = QB$: $(x+3)^2 + 16 = (x-5)^2 + 16 \rightarrow x = 1$; $Q(1,\ 0)$ — 1

$QA = QB$, so $\triangle ABQ$ is isosceles — 1

**B11.** Find the length of the median through $A$ of $\triangle ABC$ with $A(5,\ 1)$, $B(1,\ 5)$ and $C(-3,\ -1)$. **[2]**

$D$ = midpoint of $BC = (-1,\ 2)$ — 1

$AD = \sqrt{36+1} = \sqrt{37}$ units — 1

**B12.** Find the point $P$ on the segment $AB$, with $A(-3,\ 5)$ and $B(7,\ -5)$, such that $AP = \frac{2}{5}AB$. **[3]**

$AP : PB = 2 : 3$ — 1

$x = \frac{2(7)+3(-3)}{5} = 1,\ y = \frac{2(-5)+3(5)}{5} = 1$ — 1

$P = (1,\ 1)$ — 1

**B13.** For what value of $p$ are the points $A(-1,\ 3)$, $B(2,\ p)$ and $C(5,\ -1)$ collinear? (Use the section formula.) **[3]**

Let $B$ divide $AC$ in $k:1$: $\dfrac{5k-1}{k+1}=2 \to k=1$ — 1

So $B$ is the midpoint of $AC$ — 1

$p=\dfrac{3+(-1)}{2}=1$ — 1

## Long answer

**B14.** The segment joining $A(-5,\ 7)$ and $B(4,\ -2)$ is trisected at $P(p,\ 4)$ and $Q(1,\ q)$, with $P$ nearer to $A$. Find $p$ and $q$. **[3]**

$P$ divides $AB$ in $1:2$: $p=\dfrac{4+2(-5)}{3}=-2$ (and $y=\dfrac{-2+14}{3}=4$ ✓) — 1

$Q$ divides $AB$ in $2:1$: $x=\dfrac{8-5}{3}=1$ ✓ — 1

$q=\dfrac{2(-2)+7}{3}=1$ — 1

**B15.** Show that $A(-2,\ -1)$, $B(4,\ 0)$, $C(3,\ 3)$ and $D(-3,\ 2)$ are the vertices of a parallelogram. Is it a rectangle? Justify. **[4]**

Midpoint of $AC=\left(\frac{1}{2},\ 1\right)$, midpoint of $BD=\left(\frac{1}{2},\ 1\right)$ — 1

The diagonals bisect each other, so $ABCD$ is a parallelogram — 1

$AC=\sqrt{25+16}=\sqrt{41}$, $BD=\sqrt{49+4}=\sqrt{53}$ — 1

Diagonals are not equal, so it is **not** a rectangle — 1

**B16.** Find the centre and the radius of the circle passing through $A(-4,\ 2)$, $B(1,\ -3)$ and $C(5,\ 5)$. **[4]**

Let the centre be $O(x,\ y)$. $OA^2=OB^2$: $8x-4y+20=-2x+6y+10 \to x-y=-1$ — 1

$OB^2=OC^2$: $-2x+6y+10=-10x-10y+50 \to x+2y=5$ — 1

Solving: $y=2$, $x=1$; centre $(1,\ 2)$ — 1

Radius $=\sqrt{25+0}=5$ units — 1

PART C · SOLUTIONS

# C Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

## Distance formula: unknowns, circles, equidistant points

| Q | Answer | Working / reason |
|---|---|---|
| C1 | **(b)** $5\sqrt{2}$ | $\sqrt{25+25}=5\sqrt{2}$. |
| C2 | **(d)** 3 | Distance from the $x$-axis $=\lvert y\rvert=3$. |
| C3 | **(d)** 8 or −4 | $(k-2)^2+64=100 \to k-2=\pm 6 \to k=8$ or $-4$. |
| C4 | **(c)** $(5,\ -2)$ | $(5,-2)$ is $\sqrt{49+9}=\sqrt{58}$ from $(-2,-5)$ and $\sqrt{9+49}=\sqrt{58}$ from $(2,5)$. |

**C5.** A circle has centre $(a+2,\ 2a-1)$ and radius 5 units. It passes through $(1,\ 2)$. Find the possible values of $a$. [2]

$(a+1)^2+(2a-3)^2=25 \to 5a^2-10a-15=0 \to a^2-2a-3=0$ 1

$(a-3)(a+1)=0 \to a=3$ or $a=-1$ 1

**C6.** Find the point on the $x$-axis which is equidistant from $A(7,\ 6)$ and $B(-3,\ 4)$. [2]

$P(x,\ 0)$: $(x-7)^2+36=(x+3)^2+16 \to -14x+85=6x+25$ 1

$x=3$: the point is $(3,\ 0)$ 1

**C7.** Show that $A(-1,\ 0)$, $B(3,\ 0)$ and $C(1,\ 2\sqrt{3})$ are the vertices of an equilateral triangle. [2]

$AB=4$, $AC=\sqrt{4+12}=4$, $BC=\sqrt{4+12}=4$ 1

All three sides equal: equilateral 1

**C8.** $A(1,\ 3)$ is equidistant from $B(4,\ p)$ and $C(p,\ 6)$. Find $p$ and the length $AB$. [2]

$9+(p-3)^2=(p-1)^2+9 \to -6p+9=-2p+1 \to p=2$ 1

$AB=\sqrt{9+1}=\sqrt{10}$ units 1

## Section formula: ratio and point of division

| Q | Answer | Working / reason |
|---|---|---|
| C9 | **(b)** $1:2$ | $\frac{6k-6}{k+1}=-2 \to 8k=4 \to k=\frac{1}{2}$: $1:2$. |
| C10 | **(a)** $1:3$ | $\frac{6k-2}{k+1}=0 \to k=\frac{1}{3}$: $1:3$. |
| C11 | **(c)** $(2,\ 1)$ | $\frac{3x-2}{4}=1 \to x=2$; $\frac{3y+5}{4}=2 \to y=1$. |
| C12 | **(a)** | $\left(\frac{0}{2},\ \frac{0}{2}\right)=(0,\ 0)$. Both true; R explains A. |

**C13.** Find the ratio in which the $y$-axis divides the segment joining $(-4,\ 7)$ and $(8,\ -5)$. Also find the point of intersection. [3]

Ratio $k:1$: point $\left(\frac{8k-4}{k+1},\ \frac{-5k+7}{k+1}\right)$ 1

On the $y$-axis $x=0 \to k=\frac{1}{2}$: the ratio is $1:2$ 1

$y=\frac{1(-5)+2(7)}{3}=3$: the point is $(0,\ 3)$ 1

**C14.** Find the ratio in which the $x$-axis divides the join of $A(4,\ -6)$ and $B(-1,\ 4)$. Also find the point of division. [3]

Ratio $k:1$: $y=\frac{4k-6}{k+1}$ 1

$y=0 \to k=\frac{3}{2}$: the ratio is $3:2$ 1

$x=\frac{3(-1)+2(4)}{5}=1$: the point is $(1,\ 0)$ 1

**C15.** The point $P(x,\ 4)$ lies on the segment joining $A(-5,\ 8)$ and $B(4,\ -10)$. Find the ratio in which $P$ divides $AB$, and find $x$. **[3]**

Ratio $k:1$: $\dfrac{-10k+8}{k+1}=4$ — 1

$-14k=-4 \rightarrow k=\frac{2}{7}$: the ratio is $2:7$ — 1

$x=\dfrac{2(4)+7(-5)}{9}=-3$ — 1

## Midpoint: median, diameter, parallelogram

| Q | Answer | Working / reason |
|---|---|---|
| C16 | **(b)** the $y$-axis | Midpoint $(0,\ 5)$ has $x=0$. |
| C17 | **(d)** $(-1,\ -4)$ | $(2\times3-7,\ 2\times(-1)-2)=(-1,\ -4)$. |
| C18 | **(c)** 5 | $D=(2,\ 0)$; $AD=\sqrt{0+25}=5$. |
| C19 | **(a)** | Midpoint of $AC=(3.5,\ 4)=$ midpoint of $BD$ gives $D(3,\ 5)$. Both true; R explains A. |

**C20.** $ABCD$ is a parallelogram with $A(-1,\ 3)$, $B(4,\ 5)$ and $D(1,\ -2)$. Find the vertex $C$. **[2]**

Midpoint of $AC$ = midpoint of $BD=\left(\dfrac{5}{2},\ \dfrac{3}{2}\right)$ — 1

$C=(5-(-1),\ 3-3)=(6,\ 0)$ — 1

**C21.** $A(0,\ 0)$, $B(8,\ 0)$, $C(8,\ 6)$ and $D(0,\ 6)$ form a rectangle. $P$, $Q$, $R$, $S$ are the midpoints of $AB$, $BC$, $CD$, $DA$. Show that $PQRS$ is a rhombus. **[3]**

$P(4,\ 0)$, $Q(8,\ 3)$, $R(4,\ 6)$, $S(0,\ 3)$ — 1

$PQ=QR=RS=SP=\sqrt{16+9}=5$ — 1

Diagonals $PR=6$, $QS=8$ are unequal: $PQRS$ is a rhombus (not a square) — 1

PART D · SOLUTIONS

# D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

## D1. Case Study · School Garden [4]

**(i)** How far is the rose bed from the tap? **[1]**

$TR = \sqrt{36 + 64} = 10$ m — 1

**(ii)** A lamp is to be fixed midway between the rose bed and the bench. Where is it? **[1]**

$\left(\frac{8-4}{2}, \frac{11+11}{2}\right) = (2,\ 11)$ — 1

**(iii)** A sprinkler $S$ is placed on $TR$ so that $TS : SR = 1 : 3$. Find its position. **[2]**

$S = \left(\frac{1(8)+3(2)}{4}, \frac{1(11)+3(3)}{4}\right)$ — 1

$= (3.5,\ 5)$ — 1

OR

**(iii)** Show that $T$, $R$ and $B$ form an isosceles triangle. **[2]**

$TR = 10, TB = \sqrt{36+64} = 10, RB = 12$ — 1

$TR = TB$: isosceles — 1

## D2. Case Study · Town Map [4]

**(i)** Find the length of the road $SH$. **[1]**

$SH = \sqrt{144 + 81} = 15$ km — 1

**(ii)** A bus stop is built at the midpoint of $SH$. Find its position. **[1]**

$\left(\frac{-2+10}{2}, \frac{1+10}{2}\right) = (4,\ 5.5)$ — 1

**(iii)** A metro station $M$ on $SH$ is twice as far from $S$ as from $H$. Find $M$. **[2]**

$SM : MH = 2 : 1$: $M = \left(\frac{2(10)+1(-2)}{3}, \frac{2(10)+1(1)}{3}\right)$ — 1

$= (6,\ 7)$ — 1

OR

**(iii)** The road crosses the river at $P$. In what ratio does $P$ divide $SH$? Find $P$. **[2]**

$\frac{10k-2}{k+1} = 0 \to k = \frac{1}{5}$: ratio $1 : 5$ — 1

$y = \frac{1(10)+5(1)}{6} = 2.5$: $P(0,\ 2.5)$ — 1

## D3. Case Study · Drone Delivery [4]

**(i)** How far does the drone fly from $A$ to $B$? **[1]**

$AB = \sqrt{144 + 25} = 13$ units $= 1.3$ km — 1

**(ii)** It drops a parcel at the midpoint of $AB$. Where is the parcel dropped? **[1]**

$\left(\frac{-3+9}{2}, \frac{-2+3}{2}\right) = (3,\ 0.5)$ 1

**(iii)** The drone crosses the highway at $Q$. Find the ratio $AQ : QB$ and the point $Q$. **[2]**

$\frac{3k-2}{k+1} = 0 \rightarrow k = \frac{2}{3}$: $AQ : QB = 2 : 3$ 1

$x = \frac{2(9)+3(-3)}{5} = 1.8$: $Q(1.8,\ 0)$ 1

OR

**(iii)** Find the point on $AB$ that is one quarter of the way from $A$ to $B$. **[2]**

Ratio $1 : 3$: $\left(\frac{9+3(-3)}{4}, \frac{3+3(-2)}{4}\right)$ 1

$= (0,\ -0.75)$ 1

## D4. Case Study · Treasure Hunt [4]

**(i)** How far apart are the two teams? **[1]**

$AB = \sqrt{64+36} = 10$ units 1

**(ii)** The judges stand at the midpoint of $AB$. Where are they? **[1]**

$\left(\frac{1-7}{2}, \frac{8+2}{2}\right) = (-3,\ 5)$ 1

**(iii)** Find the position of the treasure. **[2]**

$T(0,\ y)$: $1 + (y-8)^2 = 49 + (y-2)^2 \rightarrow -12y = -12$ 1

$y = 1$: $T(0,\ 1)$ 1

OR

**(iii)** The treasure is at $T(0,\ 1)$. Show that $\triangle ATB$ is a right-angled isosceles triangle. **[2]**

$TA = \sqrt{1+49} = \sqrt{50}$, $TB = \sqrt{49+1} = \sqrt{50}$ 1

$TA^2 + TB^2 = 100 = AB^2$: right-angled at $T$ and isosceles 1

# E Challenge Questions – Solutions

PART E · SOLUTIONS

Worked solutions with step marks for every question in Part E.

**E1.** $O(0,\ 0)$, $A(0,\ 6)$ and $B(8,\ 0)$ form a right triangle. Show that the midpoint $M$ of the hypotenuse $AB$ is equidistant from all three vertices. **[3]**

$M = (4,\ 3)$ 1

$MA = \sqrt{16+9} = 5$, $MB = \sqrt{16+9} = 5$, $MO = \sqrt{16+9} = 5$ 1

$MA = MB = MO$: $M$ is equidistant from $O$, $A$, $B$ 1

**E2.** Two vertices of $\triangle ABC$ are $A(-1,\ 4)$ and $B(5,\ 2)$, and its centroid is $G(0,\ -3)$. Find the third vertex $C$. **[3]**

$\frac{-1+5+x}{3} = 0 \rightarrow x = -4$ — 1

$\frac{4+2+y}{3} = -3 \rightarrow y = -15$ — 1

$C = (-4,\ -15)$ — 1

**E3.** $\triangle ABC$ has $A(2,\ 7)$, $B(-4,\ 1)$, $C(5,\ -2)$. $AD$ is a median. Find the point $P$ on $AD$ with $AP : PD = 2 : 1$, and show that $P$ also divides the median $BE$ in the ratio $2 : 1$. **[3]**

$D = (0.5,\ -0.5)$; $P = \left(\frac{2(0.5)+2}{3},\ \frac{2(-0.5)+7}{3}\right) = (1,\ 2)$ — 1

$E$ = midpoint of $CA = (3.5,\ 2.5)$ — 1

$2 : 1$ on $BE$: $\left(\frac{7-4}{3},\ \frac{5+1}{3}\right) = (1,\ 2) = P$ ✓ — 1

**E4.** The point $P(x,\ y)$ is equidistant from $A(a,\ b)$ and $B(b,\ a)$, where $a \neq b$. Prove that $x = y$. **[3]**

$(x-a)^2 + (y-b)^2 = (x-b)^2 + (y-a)^2$ — 1

$-2ax - 2by = -2bx - 2ay \rightarrow 2x(b-a) - 2y(b-a) = 0$ — 1

$(b-a)(x-y) = 0$ and $a \neq b$, so $x = y$ — 1

**E5.** $A(-3,\ 2)$, $B(a,\ -1)$, $C(5,\ b)$ and $D(2,\ 4)$ are the vertices of parallelogram $ABCD$. Find $a$ and $b$, and the lengths of its sides. **[3]**

Midpoint of $AC$ = midpoint of $BD$: $\left(1,\ \frac{2+b}{2}\right) = \left(\frac{a+2}{2},\ \frac{3}{2}\right)$ — 1

$a = 0, b = 1$ — 1

$AB = \sqrt{9+9} = 3\sqrt{2}$, $BC = \sqrt{25+4} = \sqrt{29}$ (and $CD = 3\sqrt{2}$, $DA = \sqrt{29}$) — 1

**E6.** $P$ divides the join of $A(-2,\ 1)$ and $B(7,\ 4)$ in the ratio $k : 1$, and $P$ lies on the line $x - y + 1 = 0$. Find $k$ and the point $P$. **[3]**

$P\left(\frac{7k-2}{k+1},\ \frac{4k+1}{k+1}\right)$; on the line: $(7k-2) - (4k+1) + (k+1) = 0$ — 1

$4k - 2 = 0 \rightarrow k = \frac{1}{2}$ — 1

$P = (1,\ 2)$ (check: $1 - 2 + 1 = 0$ ✓) — 1