Affiliated to CBSE, New Delhi (Affiliation No. 1030642)

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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 10

Circles

OP ⊥ PT

PA = PB

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. In every tangent question, first mark the right angles at the points of contact and the equal tangent lengths.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN CIRCLES ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **2-mark tangent length:** right angle with reason 1 · Pythagoras and the answer 1.
3. **3-mark proof:** equal tangents (or right angles) with reasons 1 · adding or the congruence 1 · conclusion 1.
4. **Theorem proof:** figure, Given, To prove and Construction 1 · the steps with reasons 1 · conclusion 1.
5. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Tangent and radius

Q	Answer	Working / reason
A1	**(a)** 0	Every line through an inside point cuts the circle twice, so no tangent is possible.

A2. A point P is 17 cm from the centre of a circle. The tangent from P to the circle is 15 cm long. Find the radius. [2]

$OT \perp PT$, so $OT^2 = OP^2 - PT^2 = 289 - 225$ — 1

$OT = \sqrt{64} = 8$ cm — 1

A3. Find the length of the tangent drawn from a point 10 cm from the centre of a circle of radius 6 cm. [2]

$PT^2 = OP^2 - r^2 = 100 - 36$ (tangent $\perp$ radius) — 1

$PT = 8$ cm — 1

A4. Two concentric circles have radii 13 cm and 5 cm. Find the length of the chord of the bigger circle that touches the smaller circle. [2]

$OP \perp AB$, so P is the midpoint of AB: $AP = \sqrt{169 - 25} = 12$ cm — 1

Chord $AB = 2 \times 12 = 24$ cm — 1

Q	Answer	Working / reason
A5	**(b)** 110°	$\angle QOR = 180^\circ - 70^\circ = 110^\circ$ (quadrilateral $OQPR$ has two right angles).
A6	**(c)** 55°	$\angle APO = 35^\circ$, so $\angle POA = 90^\circ - 35^\circ = 55^\circ$.

A7. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [2]

Let AB be a diameter and l, m the tangents at A and B. Then $l \perp AB$ and $m \perp AB$ (tangent $\perp$ radius) 1

AB cuts l and m with equal alternate angles ($90°$ each), so $l \parallel m$ 1

A8. Prove that the perpendicular to a tangent at its point of contact passes through the centre of the circle. [2]

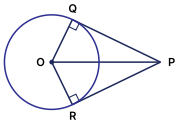
Let the tangent touch at P. The radius OP is perpendicular to the tangent at P (Theorem 1) 1

At P there is only one line perpendicular to the tangent, so this perpendicular is the line OP, which passes through O 1

Tangents from an outside point

A9. Prove that the lengths of the tangents drawn from an external point to a circle are equal. [3]

1



Given: PQ, PR tangents from P to the circle with centre O. To prove: $PQ = PR$. Construction: join OQ, OR, OP

$\angle OQP = \angle ORP = 90^\circ$ (tangent $\perp$ radius); $OQ = OR$ (radii); OP common 1

$\triangle OQP \cong \triangle ORP$ (RHS), so $PQ = PR$ (CPCT) 1

A10. A quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$. [2]

Let the circle touch AB, BC, CD, DA at P, Q, R, S. Tangents from a point are equal: $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$ 1

Adding: $(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$, so $AB + CD = AD + BC$ 1

A11. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle at the centre made by the segment joining the points of contact. [2]

Let PQ, PR be the tangents, O the centre. $\angle OQP = \angle ORP = 90^\circ$ (tangent $\perp$ radius) 1

Angle sum of quadrilateral $OQPR$: $\angle QPR + \angle QOR = 360^\circ - 180^\circ = 180^\circ$ 1

A12. Prove that a parallelogram circumscribing a circle is a rhombus. [3]

Equal tangents: $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$ 1

Adding: $AB + CD = AD + BC$ 1

$AB = CD$ and $AD = BC$ give $AB = BC$, so all sides are equal: a rhombus 1

A13. Tangents TA and TB are drawn from a point T to a circle with centre O. Show that $\angle ATB = 2\angle OAB$. [3]

Let $\angle ATB = \theta$. $TA = TB$, so $\angle TAB = \angle TBA = \dfrac{180^\circ - \theta}{2} = 90^\circ - \dfrac{\theta}{2}$ 1

$\angle OAT = 90^\circ$ (tangent $\perp$ radius), so $\angle OAB = 90^\circ - \angle TAB$ 1

$\angle OAB = \dfrac{\theta}{2}$, that is, $\angle ATB = 2\angle OAB$ 1

A14. Two parallel tangents to a circle with centre O touch it at P and Q. A third tangent touches the circle at R and meets the parallel tangents at A and B. Prove that $\angle AOB = 90^\circ$. **[3]**

Tangents from A: $\triangle OPA \cong \triangle ORA$ (RHS), so $\angle PAO = \angle RAO = \frac{1}{2}\angle PAB$. Similarly $\angle QBO = \angle RBO = \frac{1}{2}\angle QBA$ — 1

The tangents at P and Q are parallel, so $\angle PAB + \angle QBA = 180^\circ$ (co-interior angles) — 1

In $\triangle AOB$: $\angle OAB + \angle OBA = 90^\circ$, so $\angle AOB = 90^\circ$ — 1

A15. A triangle ABC is drawn to circumscribe a circle of radius 3 cm. The point of contact D divides BC into $BD = 9$ cm and $DC = 6$ cm. Find AB and AC. **[3]**

Equal tangents: $BF = 9$, $CE = 6$, $AF = AE = x$; sides $x + 9$, $x + 6$, 15; $s = x + 15$ — 1

Area $= rs = 3(x + 15)$ and, by Heron, area $= \sqrt{(x + 15) \cdot x \cdot 9 \cdot 6}$; squaring: $9(x + 15) = 54x$ — 1

$x = 3$, so $AB = 12$ cm and $AC = 9$ cm — 1

A16. PQ is a chord of length 24 cm of a circle of radius 13 cm. The tangents at P and Q meet at T. Find TP. **[3]**

OT is the perpendicular bisector of PQ at M: $PM = 12$, $OM = \sqrt{169 - 144} = 5$ cm — 1

$\triangle OMP \sim \triangle OPT$ (AA: common $\angle O$, right angles at M and P), so $\dfrac{TP}{OP} = \dfrac{PM}{OM}$ — 1

$TP = \dfrac{13 \times 12}{5} = 31.2$ cm — 1

B — Exemplar Practice – Solutions

PART B · SOLUTIONS

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(b)** 50°	$180^\circ - 130^\circ = 50^\circ$.
B2	**(c)** 55°	$\angle OAT = 90^\circ$, so $\angle BAT = 90^\circ - 35^\circ = 55^\circ$.
B3	**(b)** $6\sqrt{3}$ cm	$\angle APO = 30^\circ$, so $\tan 30^\circ = \dfrac{6}{PA}$ and $PA = 6\sqrt{3}$ cm.
B4	**(c)** 16 cm	$2\sqrt{100 - 36} = 16$ cm.
B5	**(d)** 5 cm	$OAPB$ has four right angles and $OA = OB$, so it is a square: $PA = 5$ cm.
B6	**(d)**	Every line through an inside point meets the circle twice, so A is false. R is true.
B7	**(a)**	Equal tangents give $AB + CD = AD + BC$, which forces equal sides. R explains A.

Short answer with reasons

B8. Prove that the centre of a circle that touches two intersecting lines lies on the bisector of the angle between the lines. **[2]**

Let the lines meet at P and touch the circle at A and B. $OA = OB$ (radii) and OA, OB are perpendicular to the lines — 1

$\triangle OAP \cong \triangle OBP$ (RHS, OP common), so $\angle OPA = \angle OPB$: O lies on the bisector — 1

B9. In $\triangle ABC$, $\angle B = 90^\circ$, $AB = 5$ cm and $BC = 12$ cm. Find the radius of the circle inscribed in the triangle. [2]

$AC = 13$ cm. The tangents from B are both r, so $(5 - r) + (12 - r) = 13$ 1

$r = 2$ cm 1

B10. A circle touches the side BC of $\triangle ABC$ at P, and touches AB and AC produced at Q and R. Prove that $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$). [3]

Equal tangents: $AQ = AR$, $BQ = BP$, $CP = CR$ 1

Perimeter $= AB + BC + CA = AB + (BP + PC) + CA = (AB + BQ) + (CR + CA) = AQ + AR$ 1

$= 2AQ$, so $AQ = \frac{1}{2}$ perimeter 1

B11. Tangents PA and PB are drawn from P to a circle. A tangent at a point C on the minor arc AB meets PA at E and PB at F. If $PA = 10$ cm, find the perimeter of $\triangle PEF$. [2]

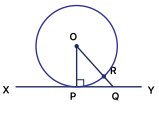
$EA = EC$ and $FB = FC$ (equal tangents), so $EF = EA + FB$ 1

Perimeter $= PE + EF + FP = (PE + EA) + (FB + FP) = PA + PB = 20$ cm 1

Long answer

B12. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. [3]

1



Given: XY touches the circle with centre O at P. To prove: $OP \perp XY$. Take any point $Q \neq P$ on XY; join OQ, cutting the circle at R

Q is outside the circle, so $OQ > OR = OP$. This holds for every $Q \neq P$ 1

OP is the shortest segment from O to XY, so $OP \perp XY$ 1

B13. AB is a diameter of a circle with centre O and AC is a chord with $\angle BAC = 30^\circ$. The tangent at C meets AB produced at D. Prove that $BC = BD$. [4]

$\angle ACB = 90^\circ$ (angle in a semicircle), so $\angle ABC = 60^\circ$ 1

$OB = OC$, so $\angle OCB = \angle OBC = 60^\circ$; $\angle OCD = 90^\circ$ (tangent $\perp$ radius), so $\angle BCD = 30^\circ$ 1

$\angle CBD = 180^\circ - 60^\circ = 120^\circ$, so $\angle BDC = 180^\circ - 120^\circ - 30^\circ = 30^\circ$ 1

$\angle BCD = \angle BDC$, so $BC = BD$ 1

B14. A circle is inscribed in $\triangle ABC$ with $AB = 12$ cm, $BC = 8$ cm and $CA = 10$ cm. It touches AB, BC, CA at D, E, F. Find AD, BE and CF. [3]

Let $AD = AF = x$, $BD = BE = y$, $CE = CF = z$: $x + y = 12$, $y + z = 8$, $z + x = 10$ 1

Adding: $x + y + z = 15$ 1

$AD = x = 7$ cm, $BE = y = 5$ cm, $CF = z = 3$ cm 1

C

PART C · SOLUTIONS

Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Tangent and radius: lengths and angles

Q	Answer	Working / reason
C1	**(a)** 32°	$\angle AOP = 180^\circ - 122^\circ = 58^\circ$ and $\angle OAP = 90^\circ$, so $\angle APO = 32^\circ$.
C2	**(b)** 45°	$\angle QPR = 180^\circ - 135^\circ = 45^\circ$.
C3	**(c)** parallel	Both are perpendicular to the diameter, so they are parallel.
C4	**(a)**	The right angle gives $OP^2 = 25 + 144 = 169$. R explains A.
C5	**(d)** 15 cm	$OP = \sqrt{81 + 144} = 15$ cm.

C6. The tangent from a point P to a circle is 20 cm long and P is 29 cm from the centre. Find the radius of the circle. [2]

Tangent $\perp$ radius, so $r^2 = 29^2 - 20^2 = 841 - 400$ — 1

$r = \sqrt{441} = 21$ cm — 1

C7. Two concentric circles have radii 17 cm and 8 cm. Find the length of the chord of the bigger circle that touches the smaller circle. [2]

The radius to the point of contact bisects the chord: half-chord $= \sqrt{289 - 64} = 15$ cm — 1

Chord $= 30$ cm — 1

C8. Tangents PA and PB are drawn from P to a circle with centre O, with $\angle APB = 60^\circ$ and $OP = 10$ cm. Find the radius and the length PA. [2]

OP bisects $\angle APB$, so $\angle APO = 30^\circ$; $OA = OP \sin 30^\circ = 5$ cm — 1

$PA = OP \cos 30^\circ = 5\sqrt{3} \approx 8.66$ cm — 1

Tangents from an outside point: proofs, polygons, incircles

C9. Prove that a rectangle circumscribing a circle is a square. [3]

Equal tangents from each vertex: $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$ — 1

Adding: $AB + CD = AD + BC$ — 1

In a rectangle $AB = CD$, $AD = BC$, so $AB = BC$: the rectangle is a square — 1

C10. A circle is inscribed in a right triangle with legs 8 cm and 15 cm. Find its radius. [3]

Hypotenuse $= \sqrt{64 + 225} = 17$ cm — 1

The tangents from the right-angle vertex are both r (a square forms), so the other tangents are $8 - r$ and $15 - r$, and $(8 - r) + (15 - r) = 17$ — 1

$r = 3$ cm — 1

C11. Tangents PA and PB are drawn from P to a circle with centre O. Prove that $\angle APB = 2\angle OAB$. **[3]**

$PA = PB$, so $\angle PAB = \angle PBA = 90^\circ - \frac{1}{2}\angle APB$ — 1

$\angle OAP = 90^\circ$, so $\angle OAB = 90^\circ - \angle PAB$ — 1

$\angle OAB = \frac{1}{2}\angle APB$, so $\angle APB = 2\angle OAB$ — 1

C12. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre. **[3]**

Let the circle (centre O) touch AB, BC, CD, DA at P, Q, R, S. By RHS, OA, OB, OC, OD bisect the angles at O: $\angle AOP = \angle AOS$, $\angle BOP = \angle BOQ$, $\angle COQ = \angle COR$, $\angle DOR = \angle DOS$ — 1

The eight angles add up to 360°, so $2(\angle AOP + \angle BOP + \angle COR + \angle DOR) = 360^\circ$ — 1

$\angle AOB + \angle COD = 180^\circ$ (and likewise $\angle BOC + \angle AOD = 180^\circ$) — 1

C13. A circle with centre O touches the sides AB, BC, CD, DA of quadrilateral $ABCD$ at Q, P, R, S. $\angle B = 90^\circ$, $AD = 17$ cm, $AB = 20$ cm and $DS = 6$ cm. Find the radius. **[3]**

$AS = AD - DS = 11$ cm, and $AQ = AS = 11$ cm (equal tangents) — 1

$BQ = AB - AQ = 9$ cm — 1

$OQBP$ is a square ($\angle B = 90^\circ$, two right angles at Q and P, $OQ = OP$), so $r = BQ = 9$ cm — 1

C14. AB is a chord of a circle with centre O and BT is the tangent at B. Prove that $\angle ABT = \frac{1}{2}\angle AOB$. **[3]**

$OA = OB$, so $\angle OBA = \dfrac{180^\circ - \angle AOB}{2} = 90^\circ - \frac{1}{2}\angle AOB$ — 1

$\angle OBT = 90^\circ$ (tangent $\perp$ radius) — 1

$\angle ABT = 90^\circ - \angle OBA = \frac{1}{2}\angle AOB$ — 1

C15. Tangents PQ and PR are drawn from an external point P. Prove that OP is the perpendicular bisector of QR. **[2]**

$PQ = PR$ (equal tangents) and $OQ = OR$ (radii) — 1

Both P and O are equidistant from Q and R, so the line OP is the perpendicular bisector of QR — 1

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Circular Running Track [4]

(i) Find the length of the path PA. **[1]**

$PA = \sqrt{900 - 225} = \sqrt{675} = 15\sqrt{3} \approx 25.98$ m — 1

(ii) Find $\angle APO$. **[1]**

$\sin \angle APO = \dfrac{15}{30} = \dfrac{1}{2}$, so $\angle APO = 30^\circ$ — 1

(iii) Show that $\triangle PAB$ is equilateral and find AB. **[2]**

$\angle APB = 2 \times 30^\circ = 60^\circ$ and $PA = PB$, so the base angles are 60° each: equilateral — 1

$AB = PA = 15\sqrt{3} \approx 25.98$ m — 1

OR

(iii) Find $\angle AOB$ and $\angle OAB$. **[2]**

$\angle AOB = 180^\circ - 60^\circ = 120^\circ$ — 1

$OA = OB$, so $\angle OAB = \dfrac{180^\circ - 120^\circ}{2} = 30^\circ$ — 1

D2. Case Study · Logo with an Incircle [4]

(i) Find AC. **[1]**

$AC = \sqrt{144 + 256} = 20$ cm — 1

(ii) Why is $BP = BQ = r$, the radius of the circle? **[1]**

$\angle B = 90^\circ$ and $OP \perp AB$, $OQ \perp BC$ with $OP = OQ$, so $OPBQ$ is a square: $BP = BQ = r$ — 1

(iii) Find the radius of the circle using tangent lengths. **[2]**

$AR = AP = 12 - r$ and $CR = CQ = 16 - r$; $AR + CR = 20$ — 1

$28 - 2r = 20$, so $r = 4$ cm — 1

OR

(iii) Find the radius by writing the area of $\triangle ABC$ in two ways. **[2]**

Area $= \frac{1}{2} \times 12 \times 16 = 96$; also area $= \frac{1}{2} r(AB + BC + CA) = 24r$ — 1

$24r = 96$, so $r = 4$ cm — 1

D3. Case Study · Fountain in a Garden [4]

(i) What is $\angle OPA$? Give a reason. **[1]**

$\angle OPA = 90^\circ$: the tangent is perpendicular to the radius at the point of contact — 1

(ii) Find the length of the path AB. **[1]**

$AP = \sqrt{169 - 25} = 12$ m, so $AB = 24$ m — 1

(iii) Two such paths touch the fountain at the two ends of one of its diameters. Show that the paths are parallel and find the distance between them. **[2]**

Both paths are perpendicular to the same diameter, so they are parallel — 1

Distance between them = the diameter = 10 m — 1

OR

(iii) A lamp post L stands on the edge of the garden. Find the length of a tangent from L to the fountain, and the total length of two straight wires along both tangents. **[2]**

$OL = 13$ m, so the tangent $= \sqrt{169 - 25} = 12$ m — 1

Two tangents are equal: total = 24 m — 1

D4. Case Study · Pulley and Belt [4]

(i) Find the length PA of each straight part. **[1]**

$PA = \sqrt{625 - 49} = 24$ cm — 1

(ii) Find the area of the quadrilateral $PAOB$. **[1]**

Area $= 2 \times \frac{1}{2} \times 7 \times 24 = 168$ cm² — 1

(iii) Find the length of AB, the chord joining the points of contact. **[2]**

$AB \perp OP$ at M; area of $\triangle OAP$: $\frac{1}{2} \cdot OP \cdot AM = \frac{1}{2} \cdot 7 \cdot 24$, so $AM = \frac{168}{25} = 6.72$ cm — 1

$AB = 2AM = 13.44$ cm — 1

OR

(iii) Prove that OP is the perpendicular bisector of AB. **[2]**

$PA = PB$ (equal tangents) and $OA = OB$ (radii) — 1

P and O are both equidistant from A and B, so OP is the perpendicular bisector of AB — 1

E PART E · SOLUTIONS
Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. Prove that the segment joining the points of contact of two parallel tangents to a circle passes through the centre. **[3]**

Let the parallel tangents touch at P and Q. Draw OR through O parallel to the tangents — 1

$\angle OPA = 90^\circ$, so $\angle POR = 90^\circ$ (co-interior angles); similarly $\angle QOR = 90^\circ$ — 1

$\angle POR + \angle QOR = 180^\circ$, so P, O, Q lie on one line — 1

E2. Two circles with centres O and O', of radii 6 cm and 8 cm, meet at P and Q so that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ. **[3]**

$OP \perp O'P$ (each is a radius meeting the other circle's tangent), so $OO' = \sqrt{36 + 64} = 10$ cm — 1

OO' bisects PQ at right angles at M; in right $\triangle OPO'$, $PM \cdot OO' = OP \cdot O'P$ — 1

$PM = \frac{48}{10} = 4.8$ cm, so $PQ = 9.6$ cm — 1

E3. An isosceles triangle ABC with $AB = AC$ circumscribes a circle. Prove that the point of contact P on BC bisects BC. **[3]**

Let the circle touch AB at Q and AC at R. Equal tangents: $AQ = AR$, $BQ = BP$, $CR = CP$ — 1

$AB = AC$ gives $AQ + BQ = AR + CR$, so $BQ = CR$ — 1

Hence $BP = CP$: P bisects BC — 1

E4. Find the radius of the circle inscribed in a triangle with sides 13 cm, 14 cm and 15 cm. **[3]**

$s = 21$; area $= \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$ cm² — 1

Area $= r \cdot s$ (three triangles with height r) — 1

$r = \dfrac{84}{21} = 4$ cm — 1

E5. Tangents PQ and PR are drawn to a circle with centre O, and $\angle QPR = 46^\circ$. S is a point on the major arc QR. Find $\angle QSR$. **[2]**

$\angle QOR = 180^\circ - 46^\circ = 134^\circ$ — 1

$\angle QSR = \frac{1}{2}\angle QOR = 67^\circ$ (angle at the circumference) — 1

E6. From a point P, 26 cm from the centre O of a circle of radius 10 cm, tangents PA and PB are drawn. Find the length of the chord AB. **[3]**

$PA = \sqrt{676 - 100} = 24$ cm — 1

OP meets AB at right angles at M; area of $\triangle OAP$: $\frac{1}{2} \cdot 26 \cdot AM = \frac{1}{2} \cdot 10 \cdot 24$ — 1

$AM = \dfrac{240}{26} = \dfrac{120}{13}$ cm, so $AB = \dfrac{240}{13} \approx 18.46$ cm — 1