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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 4

Quadratic Equations

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check every root by putting it back into the equation. In a word problem, check the answer against the story too.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN QUADRATIC EQUATIONS ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Solve by factorisation (2 marks):** correct splitting and factors 1 · both roots 1.
3. **Quadratic formula (2 marks):** value of D 1 · both roots 1.
4. **Equal roots / find k (2–3 marks):** D written and put equal to 0 1 · value(s) of k 1 · roots, if asked, 1.
5. **Word problem (5 marks):** "let" and expressions 1 · the equation 1 · standard form 1 · solving 1 · the answer, with the rejected root, 1.
6. Lines with no number are working that earns no separate mark but must be shown.

A

PART A · SOLUTIONS

NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Is it a quadratic equation?

A1. Check whether each equation is a quadratic equation: **(i)** $(x+2)^2 = 3(x-1)$ **(ii)** $x(2x+3) = 2x^2+5$ **(iii)** $(x-1)^3 = x^3+2$ **[3]**

(i) $x^2+4x+4 = 3x-3 \to x^2+x+7=0$: **quadratic** 1

(ii) $2x^2+3x = 2x^2+5 \to 3x-5=0$: **not quadratic** 1

(iii) $x^3-3x^2+3x-1 = x^3+2 \to -3x^2+3x-3=0$, i.e. $x^2-x+1=0$: **quadratic** 1

A2. Write each situation as a quadratic equation in standard form: **(i)** The product of two consecutive positive odd integers is 143. **(ii)** The length of a rectangular plot is 3 m more than twice its breadth, and its area is 230 m². **[2]**

(i) Smaller integer x: $x(x+2) = 143 \to x^2+2x-143=0$ 1

(ii) Breadth x m, length $(2x+3)$ m: $x(2x+3) = 230 \to 2x^2+3x-230=0$ 1

Solving by factorisation

A3. Find the roots by factorisation: **(i)** $x^2-5x-14=0$ **(ii)** $3x^2-x-10=0$ **(iii)** $4x^2-12x+9=0$ **[3]**

(i) $(x-7)(x+2)=0 \to x = 7$ or -2 1

(ii) $ac = -30$: $3x^2-6x+5x-10 = 3x(x-2)+5(x-2) = (3x+5)(x-2)=0 \to x = 2$ or $-\frac{5}{3}$ 1

(iii) $(2x-3)^2=0 \to x = \frac{3}{2}, \frac{3}{2}$ 1

A4. Solve by factorisation: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$. [2]

$ac = 21$: split $10x = 7x + 3x$: $\sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} = x(\sqrt{3}x + 7) + \sqrt{3}(\sqrt{3}x + 7)$ 1

$(x + \sqrt{3})(\sqrt{3}x + 7) = 0 \to x = -\sqrt{3}$ or $-\dfrac{7}{\sqrt{3}} = -\dfrac{7\sqrt{3}}{3}$ 1

A5. Solve by factorisation: **(i)** $25x^2 - 30x + 9 = 0$ **(ii)** $x^2 - (1 + \sqrt{3})x + \sqrt{3} = 0$ [2]

(i) $(5x - 3)^2 = 0 \to x = \frac{3}{5}, \frac{3}{5}$ 1

(ii) $x^2 - x - \sqrt{3}x + \sqrt{3} = x(x - 1) - \sqrt{3}(x - 1) = (x - 1)(x - \sqrt{3}) = 0 \to x = 1$ or $\sqrt{3}$ 1

A6. Find two numbers whose sum is 31 and whose product is 234. [3]

Let one number be x; the other is $31 - x$. $x(31 - x) = 234 \to x^2 - 31x + 234 = 0$ 1

$(x - 13)(x - 18) = 0 \to x = 13$ or 18 1

The numbers are 13 and 18 1

A7. Find two consecutive positive integers the sum of whose squares is 481. [3]

Let them be x and $x + 1$: $x^2 + (x + 1)^2 = 481 \to 2x^2 + 2x - 480 = 0 \to x^2 + x - 240 = 0$ 1

$(x + 16)(x - 15) = 0 \to x = 15$ (reject -16, not positive) 1

The integers are 15 and 16 1

A8. The two shorter sides of a right triangle differ by 1 cm and its hypotenuse is 29 cm. Find the two sides. [3]

Let the sides be x and $x + 1$: $x^2 + (x + 1)^2 = 29^2 \to x^2 + x - 420 = 0$ 1

$(x + 21)(x - 20) = 0 \to x = 20$ (a side cannot be -21) 1

The sides are 20 cm and 21 cm 1

A9. A small unit makes baskets. On one day, the cost of making each basket (in ₹) was 3 more than twice the number of baskets made, and the total cost was ₹90. Find the number of baskets made and the cost of each. [3]

Let x baskets be made; cost of each $= ₹(2x + 3)$. $x(2x + 3) = 90 \to 2x^2 + 3x - 90 = 0$ 1

$ac = -180$: $2x^2 + 15x - 12x - 90 = x(2x + 15) - 6(2x + 15) = (x - 6)(2x + 15) = 0 \to x = 6$ 1

6 baskets, ₹15 each (reject $x = -7.5$) 1

Quadratic formula and nature of roots

A10. Find the nature of the roots. If real roots exist, find them: **(i)** $2x^2 - 3x + 5 = 0$ **(ii)** $3x^2 - 4\sqrt{3}x + 4 = 0$ **(iii)** $2x^2 - 6x + 3 = 0$ [3]

(i) $D = 9 - 40 = -31 < 0$: **no real roots** 1

(ii) $D = 48 - 48 = 0$: equal roots, each $\dfrac{4\sqrt{3}}{6} = \dfrac{2}{\sqrt{3}}$ 1

(iii) $D = 36 - 24 = 12$: distinct real roots, $x = \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{3 \pm \sqrt{3}}{2}$ 1

A11. Find the value(s) of k for which each equation has two equal roots: **(i)** $3x^2 + kx + 12 = 0$ **(ii)** $kx(x - 3) + 9 = 0$ [2]

(i) $D = k^2 - 144 = 0 \to k = \pm 12$ 1

(ii) $kx^2 - 3kx + 9 = 0$: $D = 9k^2 - 36k = 9k(k - 4) = 0 \to k = 4$ ($k = 0$ is rejected: the equation would not be quadratic) 1

A12. Is it possible to design a rectangular park with perimeter 60 m and area 200 m²? If so, find its length and breadth. [3]

$l + b = 30$, so $b = 30 - l$. $l(30 - l) = 200 \to l^2 - 30l + 200 = 0$ 1

$D = 900 - 800 = 100 > 0$, so yes, it is possible 1

$(l - 20)(l - 10) = 0$: length 20 m, breadth 10 m 1

A13. Is it possible that the sum of the ages of two friends is 18 years and the product of their ages 4 years ago was 60? Justify. [2]

Ages x and $18 - x$: $(x - 4)(14 - x) = 60 \to x^2 - 18x + 116 = 0$ 1

$D = 324 - 464 = -140 < 0$: no real roots, so it is **not possible** 1

Speed, time and work

A14. A train travels 180 km at a uniform speed. If the speed had been 9 km/h more, it would have taken 1 hour less. Find the speed of the train. [3]

Let the speed be x km/h: $\dfrac{180}{x} - \dfrac{180}{x + 9} = 1 \to 180 \times 9 = x(x + 9)$ 1

$x^2 + 9x - 1620 = 0$; $D = 81 + 6480 = 6561 = 81^2 \to x = \dfrac{-9 + 81}{2} = 36$ 1

Speed 36 km/h (the other root, -45, is rejected) 1

A15. Two taps together fill a tank in 6 hours. The larger tap alone takes 5 hours less than the smaller one. Find the time each takes alone. [3]

Smaller tap x h, larger $(x - 5)$ h: $\dfrac{1}{x} + \dfrac{1}{x - 5} = \dfrac{1}{6} \to 6(2x - 5) = x(x - 5)$ 1

$x^2 - 17x + 30 = 0 \to (x - 15)(x - 2) = 0 \to x = 15$ ($x = 2$ would make the larger tap's time negative) 1

Smaller tap 15 hours, larger tap 10 hours 1

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(d)** $(\sqrt{2}x+\sqrt{3})^2 + x^2 = 3x^2 - 5x$	(d) simplifies to $(2\sqrt{6}+5)x+3=0$, which is linear. The other three keep an x^2 term.
B2	**(b)** $2x^2 - 7x + 6 = 0$	$2(4) - 14 + 6 = 0$ ✓. The others give 1, −2, −2.
B3	**(a)** 2	$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0 \to \frac{k}{2} = 1 \to k = 2$.
B4	**(d)** $x^2 + x - 5 = 0$	$D = 1 + 20 = 21 > 0$. The others: $D = 0$; $9 - 20 < 0$; $9 - 8\sqrt{2} \approx -2.3 < 0$.
B5	**(b)** $x^2 - 4x + 3\sqrt{2} = 0$	$D = 16 - 12\sqrt{2} \approx 16 - 16.97 < 0$. (a), (c) have $D > 0$; (d) has $D = 0$.
B6	**(c)** ±8	$k^2 - 64 = 0 \to k = \pm 8$.
B7	**(d)** $\sqrt{\frac{2}{3}},\ \sqrt{\frac{2}{3}}$	$D = 24 - 24 = 0$; each root $= \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} = \sqrt{\frac{2}{3}}$.

Short answer with reasons

B8. True or false: "If a and c have opposite signs in $ax^2 + bx + c = 0$, the equation has real roots." Justify. **[2]**

- $ac < 0$, so $-4ac > 0$ and $D = b^2 - 4ac > 0$ — 1
- **True:** the roots are real and distinct — 1

B9. Does there exist a quadratic equation with rational coefficients whose roots are both irrational? Give an example. **[2]**

- Yes. For example $x^2 - 2 = 0$ (coefficients 1, 0, −2) — 1
- Its roots are $\pm\sqrt{2}$, both irrational — 1

B10. If $x = -2$ is a root of $2x^2 + kx - 6 = 0$, find k and the other root. **[2]**

- $8 - 2k - 6 = 0 \to k = 1$ — 1
- $2x^2 + x - 6 = (2x - 3)(x + 2) = 0 \to$ other root $\frac{3}{2}$ — 1

B11. Use the quadratic formula to solve: **(i)** $2x^2 + x - 4 = 0$ **(ii)** $x^2 + 2\sqrt{2}x - 6 = 0$ **[3]**

- (i) $a = 2, b = 1, c = -4$: $D = 1 + 32 = 33$ — ½
- $x = \dfrac{-1 \pm \sqrt{33}}{4}$ — 1
- (ii) $D = 8 + 24 = 32$, $\sqrt{32} = 4\sqrt{2}$ — ½
- $x = \dfrac{-2\sqrt{2} \pm 4\sqrt{2}}{2} \to x = \sqrt{2}$ or $-3\sqrt{2}$ — 1

B12. Find the values of k for which $(k+1)x^2 - 2(k-1)x + 1 = 0$ has equal roots. Find the roots in each case. [3]

$D = 4(k-1)^2 - 4(k+1) = 4(k^2 - 3k) = 0$ 1

$k = 0$ or $k = 3$ 1

$k = 0$: $x^2 + 2x + 1 = 0 \to x = -1, -1$. $k = 3$: $4x^2 - 4x + 1 = 0 \to x = \frac{1}{2}, \frac{1}{2}$ 1

Long answer

B13. A natural number, when increased by 9, becomes 90 times its reciprocal. Find the number. [3]

$x + 9 = \dfrac{90}{x} \to x^2 + 9x - 90 = 0$ 1

$(x+15)(x-6) = 0 \to x = 6$ or -15 1

A natural number is positive: the number is 6 1

B14. A motor boat goes 48 km upstream and back. Its speed in still water is 20 km/h, and the trip upstream takes 1 hour more than the trip back. Find the speed of the stream. [3]

Stream v km/h: $\dfrac{48}{20-v} - \dfrac{48}{20+v} = 1 \to 96v = 400 - v^2$ 1

$v^2 + 96v - 400 = 0 \to (v+100)(v-4) = 0$ 1

Speed of the stream $= 4$ km/h 1

B15. The length of a rectangular field is 7 m more than its breadth, and its diagonal is 1 m more than its length. Find the length, breadth and diagonal. [3]

Breadth x, length $x+7$, diagonal $x+8$: $x^2 + (x+7)^2 = (x+8)^2$ 1

$x^2 - 2x - 15 = 0 \to (x-5)(x+3) = 0 \to x = 5$ 1

Breadth 5 m, length 12 m, diagonal 13 m 1

B16. ₹6,000 is shared equally among some people. If there were 5 more people, each would get ₹40 less. How many people are there? [3]

$\dfrac{6000}{x} - \dfrac{6000}{x+5} = 40 \to 6000 \times 5 = 40x(x+5)$ 1

$x^2 + 5x - 750 = 0 \to (x+30)(x-25) = 0$ 1

25 people 1

C · Board Pattern Practice – Solutions

PART C · SOLUTIONS

Worked solutions with step marks for every question in Part C.

Standard form · is it quadratic?

Q	Answer	Working / reason
C1	**(b)** $x^3 - x^2 = (x-1)^3$	$x^3 - x^2 = x^3 - 3x^2 + 3x - 1 \to 2x^2 - 3x + 1 = 0$. (a) is not a polynomial equation; (c) and (d) become linear.
C2	**(b)** -2	$x^2 - x - 2 = 0$: $1 - 1 - 2 = -2$.

A root is given

Q	Answer	Working / reason
C3	(a) 2	$12 - 14 + p = 0 \to p = 2$.
C4	(c) 3	$2a + 3 = 0 \to a = -\frac{3}{2}$; $2 + b = 0 \to b = -2$; $ab = 3$.

C5. If 4 is a root of $x^2 + px - 4 = 0$ and the equation $x^2 + px + k = 0$ has equal roots, find k. [2]

$16 + 4p - 4 = 0 \to p = -3$ — 1

$x^2 - 3x + k = 0$: $D = 9 - 4k = 0 \to k = \frac{9}{4}$ — 1

Discriminant · nature of roots · find k

Q	Answer	Working / reason
C6	(a) 49	$D = 25 - 4(3)(-2) = 25 + 24 = 49$.
C7	(d) $4 - 4p^2$	$D = 2^2 - 4(p)(p) = 4 - 4p^2$.
C8	(c) no real roots	$D = 16 - 24 = -8 < 0$.
C9	(b) $\frac{q^2}{4p}$	$q^2 - 4pr = 0 \to r = \frac{q^2}{4p}$.
C10	(d) $k \le 4$	$D = 16 - 4k \ge 0 \to k \le 4$.
C11	(c) a = c	Product of the roots $= \frac{c}{a}$ (Chapter 2). For roots α and $\frac{1}{\alpha}$ the product is 1, so $c = a$.
C12	(a)	$D = 9 - 20 = -11 < 0$. Both are true and R explains A.
C13	(d)	$D = 8$, not a perfect square, so the roots $1 \pm \sqrt{2}$ are irrational. A is false, R is true.

C14. Find the values of k for which $9x^2 + 3kx + 4 = 0$ has equal roots. [2]

$D = 9k^2 - 4(9)(4) = 9k^2 - 144$ — 1

$9k^2 = 144 \to k = \pm 4$ — 1

C15. Find the nature of the roots of $2x^2 - 5x + 3 = 0$. If they are real, find them. [2]

$D = 25 - 24 = 1 > 0$: two distinct real roots — 1

$x = \frac{5 \pm 1}{4} \to x = \frac{3}{2}$ or 1 — 1

C16. Find k for which $4x^2 - kx + 9 = 0$ has equal roots. Find the roots for each value of k. [3]

$D = k^2 - 144 = 0$ — 1

$k = 12$ or $k = -12$ — 1

$k = 12$: $(2x - 3)^2 = 0 \to x = \frac{3}{2}, \frac{3}{2}$; $k = -12$: $(2x + 3)^2 = 0 \to x = -\frac{3}{2}, -\frac{3}{2}$ — 1

C17. Find k ($k \neq 2$) for which $(k - 2)x^2 + 2(k - 2)x + 4 = 0$ has equal roots. Also find the roots. [3]

$D = 4(k - 2)^2 - 16(k - 2) = 4(k - 2)(k - 6) = 0$ — 1

$k \neq 2$, so $k = 6$ — 1

$4x^2 + 8x + 4 = 0 \to (x + 1)^2 = 0 \to x = -1, -1$ — 1

Solve: factorisation or formula

C18. Solve by factorisation: $6x^2 - 7x - 3 = 0$. [2]

$ac = -18$: $6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3) = 0$ 1

$x = \frac{3}{2}$ or $-\frac{1}{3}$ 1

C19. Solve using the quadratic formula: $x^2 - 4x - 1 = 0$. [2]

$D = 16 + 4 = 20$, $\sqrt{20} = 2\sqrt{5}$ 1

$x = \dfrac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$ 1

C20. Solve: $\dfrac{1}{x} - \dfrac{1}{x+2} = \dfrac{1}{4}$, $x \neq 0, -2$. [2]

$\dfrac{2}{x(x+2)} = \dfrac{1}{4} \rightarrow x^2 + 2x - 8 = 0$ 1

$(x+4)(x-2) = 0 \rightarrow x = 2$ or -4 1

C21. Solve for x: $x^2 - 4ax + 4a^2 - 9b^2 = 0$. [2]

$(x-2a)^2 = (3b)^2$, i.e. $(x - 2a - 3b)(x - 2a + 3b) = 0$ 1

$x = 2a + 3b$ or $x = 2a - 3b$ 1

Word problems

C22. The product of two consecutive positive even integers is 168. Find them. [2]

$x(x+2) = 168 \rightarrow x^2 + 2x - 168 = 0 \rightarrow (x+14)(x-12) = 0$ 1

$x = 12$ (positive): the integers are 12 and 14 1

C23. The product of Ravi's age 5 years ago and his age 3 years from now is 105. Find his present age. [3]

Present age x: $(x-5)(x+3) = 105 \rightarrow x^2 - 2x - 120 = 0$ 1

$(x-12)(x+10) = 0 \rightarrow x = 12$ or -10 1

Age cannot be negative: Ravi is 12 years old 1

C24. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{1}{6}$. Find the numbers. [3]

Smaller x, larger $x+3$: $\dfrac{1}{x} - \dfrac{1}{x+3} = \dfrac{1}{6} \rightarrow \dfrac{3}{x(x+3)} = \dfrac{1}{6}$ 1

$x^2 + 3x - 18 = 0 \rightarrow (x+6)(x-3) = 0 \rightarrow x = 3$ 1

The numbers are 3 and 6 1

C25. The perimeter of a right triangle is 30 cm and its hypotenuse is 13 cm. Find the other two sides. [3]

Sides a and $17 - a$: $a^2 + (17-a)^2 = 169 \rightarrow 2a^2 - 34a + 120 = 0$ 1

$a^2 - 17a + 60 = 0 \rightarrow (a-5)(a-12) = 0$ 1

The sides are 5 cm and 12 cm 1

C26. Ria and Sid together have 40 stickers. Each loses 4 stickers, and the product of the numbers they now have is 240. How many stickers did each have at the start? **[3]**

Ria x, Sid $40-x$: $(x-4)(36-x)=240 \to x^2-40x+384=0$ — 1

$D=1600-1536=64 \to x=\dfrac{40\pm 8}{2}=24$ or 16 — 1

One had 24 and the other 16 stickers — 1

C27. A rectangular park 30 m long and 20 m wide has a path of uniform width running along the inside of its boundary; the rest is lawn. The area of the path is 184 m². Find the width of the path. **[3]**

Width w m. Lawn: $(30-2w)(20-2w)=600-184=416$ — 1

$4w^2-100w+184=0 \to w^2-25w+46=0 \to (w-2)(w-23)=0$ — 1

$w=23$ is too wide (more than half of 20 m), so the path is 2 m wide — 1

C28. An express train takes 1 hour less than a passenger train to cover 300 km. The express train is 10 km/h faster. Find the speeds of the two trains. **[5]**

Let the passenger train's speed be x km/h; the express train's is $(x+10)$ km/h. — ½

Times: $\dfrac{300}{x}$ h and $\dfrac{300}{x+10}$ h — 1

$\dfrac{300}{x}-\dfrac{300}{x+10}=1 \to 3000=x(x+10)$ — 1

$x^2+10x-3000=0$ — 1

$(x+60)(x-50)=0 \to x=50$ or -60 — 1

Speeds: passenger train 50 km/h, express train 60 km/h (reject -60) — ½

C29. A plane left 30 minutes late. To reach its destination 1,200 km away on time, it flew 200 km/h faster than usual. Find its usual speed. **[5]**

Let the usual speed be x km/h. — ½

$\dfrac{1200}{x}-\dfrac{1200}{x+200}=\dfrac{1}{2}$ — 1

$2\times 1200\times 200=x(x+200) \to x^2+200x-480000=0$ — 1

$D=40000+1920000=1960000$, $\sqrt{D}=1400$ — 1

$x=\dfrac{-200\pm 1400}{2}=600$ or -800 — 1

Usual speed 600 km/h — ½

C30. The sum of the areas of two squares is 260 m² and the difference of their perimeters is 24 m. Find the sides of the two squares. **[5]**

Let the smaller side be x m. $4(\text{larger})-4x=24$, so the larger side is $(x+6)$ m. — 1

$x^2+(x+6)^2=260$ — 1

$2x^2+12x-224=0 \to x^2+6x-112=0$ — 1

$(x+14)(x-8)=0 \to x=8$ (a side cannot be -14) — 1

The sides are 8 m and 14 m — 1

C31. A shopkeeper buys some books for ₹1,200. If each book cost ₹20 less, he could buy 5 more books for the same money. How many books did he buy? [3]

$\frac{1200}{x} - \frac{1200}{x+5} = 20 \rightarrow 6000 = 20x(x+5)$ 1

$x^2 + 5x - 300 = 0 \rightarrow (x+20)(x-15) = 0$ 1

He bought 15 books 1

D — Case-Based Questions – Solutions

PART D · SOLUTIONS

Worked solutions with step marks for every question in Part D.

D1. Case Study · Diwali Mela Stall [4]

(i) Write the cost of one diya in terms of x. [1]

$₹\frac{600}{x}$ 1

(ii) Form a quadratic equation in x in standard form. [1]

$\frac{600}{x} - \frac{600}{x+10} = 5 \rightarrow 6000 = 5x(x+10) \rightarrow x^2 + 10x - 1200 = 0$ 1

(iii) Find the number of diyas she bought. [2]

$(x+40)(x-30) = 0$ 1

$x = 30$ (reject -40): she bought 30 diyas 1

OR

(iii) She sells all the diyas at ₹25 each. Find her profit. [2]

From the equation, $x = 30$; cost of one diya $= \frac{600}{30} = ₹20$ 1

Profit $= 30 \times (25 - 20) = ₹150$ 1

D2. Case Study · Ball in the Air [4]

(i) Find the height of the ball after 1 second. [1]

$h = 20 - 5 = 15$ m 1

(ii) Using the discriminant, check whether the ball can reach a height of 25 m. [1]

$5t^2 - 20t + 25 = 0 \rightarrow t^2 - 4t + 5 = 0$; $D = 16 - 20 < 0$: no real t, so it **cannot** reach 25 m 1

(iii) At what times is the ball 15 m above the ground? [2]

$5t^2 - 20t + 15 = 0 \rightarrow t^2 - 4t + 3 = 0$ 1

$(t-1)(t-3) = 0$: at $t = 1$ s (going up) and $t = 3$ s (coming down) 1

OR

(iii) Find the height h for which $5t^2 - 20t + h = 0$ has equal roots. What does this height mean? [2]

$D = 400 - 20h = 0 \rightarrow h = 20$ 1

It is reached only once (at $t = 2$ s): 20 m is the greatest height 1

D3. Case Study · School Garden [4]

(i) Taking the breadth as x m, write the quadratic equation. **[1]**

$x(x+4) = 96 \rightarrow x^2 + 4x - 96 = 0$ 1

(ii) Find the discriminant of this equation. **[1]**

$D = 16 + 384 = 400$ 1

(iii) Find the length and breadth of the garden. **[2]**

$x = \dfrac{-4 \pm 20}{2} = 8$ (reject -12) 1

Breadth 8 m, length 12 m 1

OR

(iii) Fencing costs ₹50 per metre. Find the cost of fencing the garden. **[2]**

Garden 12 m by 8 m (from the equation); perimeter $= 2(12+8) = 40$ m 1

Cost $= 40 \times 50 =$ ₹2,000 1

D4. Case Study · Road Trip [4]

(i) Write the time taken for the onward journey in terms of x. **[1]**

$\dfrac{150}{x}$ hours 1

(ii) Form the quadratic equation in standard form. **[1]**

$\dfrac{150}{x-10} - \dfrac{150}{x} = \dfrac{1}{2} \rightarrow 3000 = x(x-10) \rightarrow x^2 - 10x - 3000 = 0$ 1

(iii) Find his speed on the onward journey. **[2]**

$(x-60)(x+50) = 0$ 1

$x = 60$: onward speed 60 km/h 1

OR

(iii) Find the total time for the round trip. **[2]**

Onward speed 60 km/h, return 50 km/h ½

$\dfrac{150}{60} + \dfrac{150}{50} = 2.5 + 3$ 1

$= 5.5$ hours ½

E PART E · SOLUTIONS
Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. If the roots of $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$. [3]

$D = (b-c)^2 - 4(a-b)(c-a) = 0$ 1

Expanding: $4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$, i.e. $(2a-b-c)^2 = 0$ 1

So $2a - b - c = 0$, i.e. $2a = b + c$ 1

E2. Prove that $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$ has no real roots if $ad \neq bc$. **[3]**

$D = 4(ac+bd)^2 - 4(a^2+b^2)(c^2+d^2)$ — 1

$= 4[2abcd - a^2d^2 - b^2c^2] = -4(ad-bc)^2$ — 1

If $ad \neq bc$, $D < 0$, so there are no real roots — 1

E3. Find k for which $x^2-2(k+1)x+(k+7)=0$ has equal roots. Find the roots in each case. **[3]**

$\frac{D}{4} = (k+1)^2 - (k+7) = k^2+k-6 = 0$ — 1

$(k+3)(k-2)=0 \rightarrow k = 2$ or -3 — 1

$k=2$: $x^2-6x+9=0 \rightarrow x=3,3$; $\quad k=-3$: $x^2+4x+4=0 \rightarrow x=-2,-2$ — 1

E4. Solve for x: $abx^2+(b^2-ac)x-bc=0$ $(a,b\neq 0)$. **[2]**

$abx^2+b^2x-acx-bc = bx(ax+b)-c(ax+b) = (bx-c)(ax+b)=0$ — 1

$x = \frac{c}{b}$ or $x = -\frac{b}{a}$ — 1

E5. Working alone, A takes 6 days less than B to finish a job. Together they finish it in 4 days. How long would each take alone? **[3]**

A x days, B $(x+6)$ days: $\frac{1}{x}+\frac{1}{x+6}=\frac{1}{4} \rightarrow 4(2x+6)=x(x+6)$ — 1

$x^2-2x-24=0 \rightarrow (x-6)(x+4)=0 \rightarrow x=6$ — 1

A alone 6 days, B alone 12 days — 1

E6. The product of the digits of a two-digit number is 14. When 45 is subtracted from the number, the digits change places. Find the number. **[3]**

Tens digit x, units y: $xy=14$ and $10x+y-45=10y+x \rightarrow x-y=5$ — 1

$y(y+5)=14 \rightarrow y^2+5y-14=0 \rightarrow (y+7)(y-2)=0 \rightarrow y=2$ — 1

$x=7$: the number is 72 (check: $72-45=27$ ✓) — 1

E7. If $x^2+2px+mn=0$ has equal real roots, show that $x^2-2(m+n)x+(m^2+n^2+2p^2)=0$ also has equal roots. **[3]**

First equation: $4p^2-4mn=0$, so $p^2=mn$ — 1

Second: $\frac{D}{4} = (m+n)^2-(m^2+n^2+2p^2) = 2mn-2p^2$ — 1

$= 2mn-2mn=0$, so its roots are equal too — 1