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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 2

Polynomials

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check a zero by putting it back in $p(x)$: the value must be 0.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN POLYNOMIALS ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Find the zeroes (2 marks):** splitting / factorising 1 · both zeroes 1.
3. **Verify the relationship:** sum $= -\frac{b}{a}$ ½ · product $= \frac{c}{a}$ ½, each with both sides worked out.
4. **Expression in α, β:** sum and product from the coefficients 1 · identity and substitution 1 · answer as required.
5. **Form a polynomial:** $x^2 - Sx + P$ with values put in 1 · simplified (fractions cleared) 1.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Zeroes from a graph

A1. The graphs of $y = p(x)$ are given below. Find the number of zeroes of $p(x)$ in each case. **[2]**

(i) The line meets the x-axis at one point → **1** zero. ½

(ii) The curve does not meet the x-axis → **0** zeroes. ½

(iii) The curve meets the x-axis at three points → **3** zeroes. ½

(iv) The curve touches the x-axis at one point → **1** zero. ½

Q	Answer	Working / reason
A2	**(c)** 3	Each point where the graph meets the x-axis gives one zero: 3 points → 3 zeroes.
A3	**(b)** –1 and 3	The curve meets the x-axis at $x = -1$ and $x = 3$. The point $(0, -3)$ is on the y-axis, so -3 is not a zero.

Find the zeroes and verify the relationship

A4. Find the zeroes of $x^2 - 3x - 10$ and verify the relationship between the zeroes and the coefficients. **[2]**

$x^2 - 5x + 2x - 10 = x(x-5) + 2(x-5) = (x-5)(x+2)$; zeroes 5 and -2 1

Sum $= 5 + (-2) = 3 = -\dfrac{-3}{1} = -\dfrac{b}{a}$ ✓ ½

Product $= 5 \times (-2) = -10 = \dfrac{-10}{1} = \dfrac{c}{a}$ ✓ ½

A5. Find the zeroes of $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients. **[2]**

$4s^2 - 4s + 1 = (2s-1)^2$; zeroes $\frac{1}{2}$ and $\frac{1}{2}$ — 1

Sum $= \frac{1}{2} + \frac{1}{2} = 1 = -\frac{-4}{4}$ ✓ — ½

Product $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{c}{a}$ ✓ — ½

A6. Find the zeroes of $8x^2 - 2x - 3$ and verify the relationship between the zeroes and the coefficients. **[3]**

$ac = -24$: numbers -6 and 4. $8x^2 - 6x + 4x - 3 = 2x(4x-3) + 1(4x-3)$ — 1

$= (4x-3)(2x+1)$; zeroes $\frac{3}{4}$ and $-\frac{1}{2}$ — 1

Sum $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4} = -\frac{-2}{8}$ ✓ — ½

Product $= -\frac{3}{8} = \frac{-3}{8}$ ✓ — ½

A7. Find the zeroes of $5u^2 + 15u$ and verify the relationship between the zeroes and the coefficients. **[2]**

$5u^2 + 15u = 5u(u+3)$; zeroes 0 and -3 — 1

Sum $= -3 = -\frac{15}{5}$ ✓; Product $= 0 = \frac{0}{5}$ ✓ (here $c = 0$) — 1

A8. Find the zeroes of $y^2 - 18$ and verify the relationship between the zeroes and the coefficients. **[2]**

$y^2 - 18 = (y - 3\sqrt{2})(y + 3\sqrt{2})$; zeroes $3\sqrt{2}$ and $-3\sqrt{2}$ — 1

Sum $= 0 = -\frac{0}{1}$ ✓; Product $= -18 = \frac{-18}{1}$ ✓ — 1

A9. Find the zeroes of $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients. **[3]**

$ac = -12$: numbers -4 and 3. $3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4)$ — 1

$= (3x-4)(x+1)$; zeroes $\frac{4}{3}$ and -1 — 1

Sum $= \frac{4}{3} - 1 = \frac{1}{3} = -\frac{-1}{3}$ ✓ — ½

Product $= -\frac{4}{3} = \frac{-4}{3}$ ✓ — ½

Form a quadratic polynomial

A10. Find a quadratic polynomial, the sum and product of whose zeroes are given: **(i)** $5,\ 6$ **(ii)** $-\frac{2}{3},\ -\frac{1}{3}$ **(iii)** $\sqrt{3},\ -1$ **[3]**

(i) $x^2 - 5x + 6$ — 1

(ii) $x^2 + \frac{2}{3}x - \frac{1}{3}$; multiply by $3 \to 3x^2 + 2x - 1$ — 1

(iii) $x^2 - \sqrt{3}\,x - 1$ — 1

A11. Find a quadratic polynomial, the sum and product of whose zeroes are given: **(i)** $0,\ \sqrt{7}$ **(ii)** $\frac{1}{4},\ -1$ **(iii)** $4,\ 4$ **[3]**

(i) $x^2 - 0 \cdot x + \sqrt{7} = x^2 + \sqrt{7}$ — 1

(ii) $x^2 - \frac{1}{4}x - 1$; multiply by $4 \to 4x^2 - x - 4$ — 1

(iii) $x^2 - 4x + 4$ — 1

A12. One zero of a quadratic polynomial is -2 and the sum of its zeroes is 1. Find the polynomial. **[2]**

Other zero $= 1 - (-2) = 3$ — ½

Product $= (-2)(3) = -6$ — ½

Polynomial: $x^2 - x - 6$ — 1

Use the relationship

Q	Answer	Working / reason
A13	**(b)** 8	$p(2) = 4 - 12 + k = 0 \Rightarrow k = 8$

A14. If α and β are the zeroes of $x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[2]**

$\alpha + \beta = 5,\ \alpha\beta = 4$ — ½

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5}{4}$ — ½

Value $= \frac{5}{4} - 2(4) = \frac{5}{4} - 8 = -\frac{27}{4}$ — 1

A15. Verify that 3 and $-\frac{1}{2}$ are the zeroes of $2x^2 - 5x - 3$. Also verify the relationship between the zeroes and the coefficients. **[2]**

$p(3) = 18 - 15 - 3 = 0$; $p\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{5}{2} - 3 = 0$, so both are zeroes — 1

Sum $= 3 - \frac{1}{2} = \frac{5}{2} = -\frac{-5}{2}$ ✓; Product $= -\frac{3}{2} = \frac{-3}{2}$ ✓ — 1

Q	Answer	Working / reason
A16	**(a)** 0	$x^2 + 4 \geq 4$ for every x, so it is never 0. The graph lies above the x-axis.

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(b)** -2	$2\cdot\frac{1}{4}+\frac{3}{2}+k=0\Rightarrow\frac{1}{2}+\frac{3}{2}+k=0\Rightarrow k=-2$
B2	**(c)** $\frac{x^2}{2}-\frac{x}{2}-6$	Sum 1, product $-12\to x^2-x-12$. Option (c) is $\frac{1}{2}(x^2-x-12)$, which has the same zeroes.
B3	**(b)** both negative	Sum $=-99<0$ and product $=127>0\to$ both negative.
B4	**(d)** a = 0, b = –6	Sum $=-1=-(a+1)\Rightarrow a=0$; product $=-6=b$.
B5	**(c)** c and a have the same sign	Equal zeroes: $\alpha\beta=\alpha^2=\frac{c}{a}>0$ (for non-zero zeroes), so c and a have the same sign.
B6	**(a)** 3	Sum $=0\Rightarrow k-3=0\Rightarrow k=3$

Short answer with reasons

B7. State whether each statement is true or false. Give a reason. **(i)** If both zeroes of ax^2+bx+c are positive, then a, b and c all have the same sign. **(ii)** If the graph of a polynomial meets the x-axis at only one point, it cannot be a quadratic polynomial. [2]

(i) **False.** Sum $=-\frac{b}{a}>0$ means b has the opposite sign to a; product $\frac{c}{a}>0$ means c has the same sign as a. 1

(ii) **False.** $x^2-2x+1=(x-1)^2$ is quadratic and its graph touches the x-axis only at $x=1$. 1

B8. Find the zeroes of $2x^2-(1+2\sqrt{2})x+\sqrt{2}$ and verify the relationship between the zeroes and the coefficients. [3]

$2x^2-x-2\sqrt{2}x+\sqrt{2}=x(2x-1)-\sqrt{2}(2x-1)=(2x-1)(x-\sqrt{2})$ 1

Zeroes: $\frac{1}{2}$ and $\sqrt{2}$ 1

Sum $=\frac{1}{2}+\sqrt{2}=\frac{1+2\sqrt{2}}{2}=-\frac{b}{a}$ ✓ ½

Product $=\frac{\sqrt{2}}{2}=\frac{c}{a}$ ✓ ½

B9. Find the zeroes of $v^2+4\sqrt{3}\,v-15$ and verify the relationship between the zeroes and the coefficients. [3]

$v^2+5\sqrt{3}v-\sqrt{3}v-15=v(v+5\sqrt{3})-\sqrt{3}(v+5\sqrt{3})=(v-\sqrt{3})(v+5\sqrt{3})$ 1

Zeroes: $\sqrt{3}$ and $-5\sqrt{3}$ 1

Sum $=-4\sqrt{3}=-\frac{4\sqrt{3}}{1}$ ✓ ½

Product $=-15=\frac{-15}{1}$ ✓ ½

B10. Find the zeroes of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. [3]

Multiply by 3: $21y^2 - 11y - 2 = 21y^2 - 14y + 3y - 2 = 7y(3y-2) + 1(3y-2) = (7y+1)(3y-2)$ — 1

Zeroes: $-\frac{1}{7}$ and $\frac{2}{3}$ — 1

Sum $= \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$ and $-\frac{b}{a} = \frac{11/3}{7} = \frac{11}{21}$ ✓ — ½

Product $= -\frac{2}{21}$ and $\frac{c}{a} = \frac{-2/3}{7} = -\frac{2}{21}$ ✓ — ½

B11. The zeroes of $x^2 + px + q$ are double the zeroes of $2x^2 - 5x - 3$. Find p and q. [2]

$2x^2 - 5x - 3 = (2x+1)(x-3)$: zeroes $-\frac{1}{2}$ and 3 — ½

Required zeroes: -1 and 6 — ½

Sum $= 5 = -p \Rightarrow p = -5$; product $= -6 = q$ — 1

B12. If α and β are the zeroes of $x^2 - 2x - 8$, find a quadratic polynomial whose zeroes are 2α and 2β. [2]

$\alpha + \beta = 2$, $\alpha\beta = -8$ — ½

New sum $= 2(\alpha+\beta) = 4$; new product $= 4\alpha\beta = -32$ — ½

Polynomial: $x^2 - 4x - 32$ — 1

C

PART C · SOLUTIONS

Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Zeroes from a graph

Q	Answer	Working / reason
C1	**(b)** 2	The curve crosses the x-axis at $x = -2$ and touches it at $x = 1$: two distinct points → 2 zeroes.
C2	**(a)** 0	The graph never meets the x-axis. $(0, 3)$ is on the y-axis, so it is not a zero.
C3	**(b)** 2	p meets the x-axis at $-2, 1, 3$; q at $-2, 3$. Common: -2 and 3.
C4	**(b)**	A is true: $x^2 - 6x + 9 = (x-3)^2$ touches the axis at $x = 3$. R is true, but it does not explain why there is only one point.
C5	**(d)**	A is false: $y = x^2$ is quadratic and touches the x-axis only at the origin. R is true.
C6	**(b)** a < 0, c > 0	Opens down → $a < 0$. Zeroes of opposite signs → $\frac{c}{a} < 0$, so $c > 0$.

Find the zeroes (and verify)

C7. Find the zeroes of $p(x) = x^2 + \frac{1}{6}x - \frac{1}{3}$. [2]

Multiply by 6: $6x^2 + x - 2 = 6x^2 + 4x - 3x - 2 = 2x(3x+2) - 1(3x+2)$ — ½

$= (3x+2)(2x-1)$ — ½

Zeroes: $-\frac{2}{3}$ and $\frac{1}{2}$ — 1

C8. Find the zeroes of $4x^2 - 4x - 3$ and verify the relationship between the zeroes and the coefficients. [3]

$4x^2 - 6x + 2x - 3 = 2x(2x-3) + 1(2x-3) = (2x-3)(2x+1)$ — 1

Zeroes: $\frac{3}{2}$ and $-\frac{1}{2}$ — 1

Sum $= 1 = -\frac{-4}{4}$ ✓ — ½

Product $= -\frac{3}{4} = \frac{-3}{4}$ ✓ — ½

C9. Find the zeroes of $x^2 - 2\sqrt{3}\,x - 9$ and verify the relationship between the zeroes and the coefficients. [3]

$x^2 - 3\sqrt{3}x + \sqrt{3}x - 9 = x(x - 3\sqrt{3}) + \sqrt{3}(x - 3\sqrt{3}) = (x - 3\sqrt{3})(x + \sqrt{3})$ — 1

Zeroes: $3\sqrt{3}$ and $-\sqrt{3}$ — 1

Sum $= 2\sqrt{3} = -\frac{-2\sqrt{3}}{1}$ ✓ — ½

Product $= -9 = \frac{-9}{1}$ ✓ — ½

Value of an expression in α and β

Q	Answer	Working / reason
C10	**(c)** −2	$\frac{\alpha+\beta}{\alpha\beta} = \frac{4}{-2} = -2$
C11	**(c)** $\frac{16}{3}$	$(\alpha+\beta)^2 - 4\alpha\beta = 4 - 4\left(-\frac{1}{3}\right) = \frac{16}{3}$
C12	**(d)** 19	$(\alpha+\beta)^2 - 2\alpha\beta = 25 - 6 = 19$

C13. If α and β are the zeroes of $x^2 - 7x + 10$, find (i) $\alpha^2 + \beta^2$ (ii) $\alpha^2\beta + \alpha\beta^2$. [2]

$\alpha + \beta = 7$, $\alpha\beta = 10$ — ½

(i) $\alpha^2 + \beta^2 = 49 - 20 = 29$ — ½

(ii) $\alpha\beta(\alpha+\beta) = 10 \times 7 = 70$ — 1

C14. If α and β are the zeroes of $3x^2 - 7x + 2$, find (i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. [3]

$\alpha + \beta = \frac{7}{3}$, $\alpha\beta = \frac{2}{3}$ — ½

$\alpha^2 + \beta^2 = \frac{49}{9} - \frac{4}{3} = \frac{37}{9}$ — ½

(i) $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{37}{9} \times \frac{3}{2} = \frac{37}{6}$ — 1

(ii) $\frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{37}{9} \times \frac{9}{4} = \frac{37}{4}$ — 1

C15. If α and β are the zeroes of $2x^2 + 5x - 3$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

$\alpha + \beta = -\frac{5}{2}$, $\alpha\beta = -\frac{3}{2}$ — 1

New sum $= \frac{\alpha+\beta}{\alpha\beta} = \frac{-5/2}{-3/2} = \frac{5}{3}$; new product $= \frac{1}{\alpha\beta} = -\frac{2}{3}$ — 1

$x^2 - \frac{5}{3}x - \frac{2}{3}$, i.e. $3x^2 - 5x - 2$ — 1

C16. If α and β are the zeroes of $x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$. [2]

$\alpha + \beta = 1, \alpha\beta = -4$ ½

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{-4} = -\frac{1}{4}$ ½

Value $= -\frac{1}{4} - (-4) = \frac{15}{4}$ 1

C17. If α and β are the zeroes of $x^2 - 8x + k$ and $\alpha^2 + \beta^2 = 40$, find k. [2]

$\alpha + \beta = 8, \alpha\beta = k$ ½

$\alpha^2 + \beta^2 = 64 - 2k = 40$ ½

$k = 12$ 1

Find an unknown coefficient

Q	Answer	Working / reason
C18	**(b)** 3	$\frac{k\sqrt{3}}{3} = \sqrt{3} \Rightarrow k = 3$
C19	**(c)** 3	$-4 + \frac{k}{2} = -\frac{5}{2} \Rightarrow \frac{k}{2} = \frac{3}{2} \Rightarrow k = 3$
C20	**(d)** –3	Product $= 1 \Rightarrow 2k + 1 = k - 2 \Rightarrow k = -3$
C21	**(a)** 1	$p(1) = a + b + c = 0$, so $x = 1$ is a zero.

C22. If one zero of $3x^2 - 8x + 2k + 1$ is seven times the other, find the zeroes and the value of k. [3]

Let the zeroes be α and 7α. Sum: $8\alpha = \frac{8}{3} \Rightarrow \alpha = \frac{1}{3}$ 1

Zeroes: $\frac{1}{3}$ and $\frac{7}{3}$ ½

Product: $7\alpha^2 = \frac{7}{9} = \frac{2k+1}{3}$ ½

$2k + 1 = \frac{7}{3} \Rightarrow k = \frac{2}{3}$ 1

C23. If 2 is a zero of $2x^2 + kx - 6$, find k and the other zero. [2]

$p(2) = 8 + 2k - 6 = 0 \Rightarrow k = -1$ 1

Product $= \frac{-6}{2} = -3$, so other zero $= \frac{-3}{2}$ 1

C24. If one zero of $x^2 + 3x + k$ is 2, find the value of k and the other zero. [2]

$4 + 6 + k = 0 \Rightarrow k = -10$ 1

Sum $= -3$, so other zero $= -3 - 2 = -5$ 1

C25. Find a quadratic polynomial whose zeroes are each 3 more than the zeroes of $x^2 - 5x + 6$. [2]

$x^2 - 5x + 6 = (x - 2)(x - 3)$: zeroes 2 and 3 ½

New zeroes 5 and 6: sum 11, product 30 ½

Polynomial: $x^2 - 11x + 30$ 1

Form a quadratic polynomial

C26. Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find its zeroes. [3]

$p(x) = x^2 - (-3)x + 2 = x^2 + 3x + 2$ 1

$x^2 + 2x + x + 2 = (x + 2)(x + 1)$ 1

Zeroes: -2 and -1 1

C27. Find a quadratic polynomial whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$. [2]

Sum $= 4$; product $= (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$ 1

Polynomial: $x^2 - 4x + 1$ 1

Q	Answer	Working / reason
C28	**(c)** $x^2 - 25$	Other zero $= -5$; product $= -25 \rightarrow x^2 - 25$

Signs of zeroes · spot a zero

Q	Answer	Working / reason
C29	**(b)** 1 and $\frac{5}{4}$	$4 - 9 + 5 = 0$, so 1 is a zero; other zero $= \frac{c}{a} = \frac{5}{4}$.
C30	**(b)** both negative	Sum $= -7 < 0$, product $= 10 > 0 \rightarrow$ both negative (-2 and -5).
C31	**(a)**	Product $= \frac{c}{a} = -\frac{5}{2} < 0 \rightarrow$ opposite signs (zeroes 1 and $-\frac{5}{2}$). R is the reason.

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Garden Fountain [4]

(i) How many zeroes does $p(x)$ have? Write them from the graph. [1]

Two zeroes: -2 and 4 (where the curve meets the x-axis) 1

(ii) Find the sum and the product of the zeroes. [1]

Sum $= -2 + 4 = 2$; product $= (-2)(4) = -8$ 1

(iii) Verify that the sum of the zeroes $= -\frac{b}{a}$ and the product $= \frac{c}{a}$. [2]

$a = -1$, $b = 2$: $-\frac{b}{a} = -\frac{2}{-1} = 2 =$ sum ✓ 1

$c = 8$: $\frac{c}{a} = \frac{8}{-1} = -8 =$ product ✓ 1

OR

(iii) The jet is highest midway between the zeroes. Find that value of x and the greatest height of the jet. [2]

Midway: $x = \frac{-2 + 4}{2} = 1$ 1

Height $= p(1) = -1 + 2 + 8 = 9$ m 1

D2. Case Study · Water Rocket [4]

(i) What type of polynomial is $h(t)$? What is the height of the roof? **[1]**

Quadratic (degree 2). Roof height $= h(0) = 25$ m 1

(ii) Write the sum and the product of the zeroes of $h(t)$ without finding them. **[1]**

Sum $= -\dfrac{20}{-5} = 4$; product $= \dfrac{25}{-5} = -5$ 1

(iii) Find the zeroes of $h(t)$. After how many seconds does the rocket land? **[2]**

$-5(t^2 - 4t - 5) = -5(t-5)(t+1)$; zeroes 5 and -1 1

Time cannot be negative, so it lands after 5 s 1

OR

(iii) Find the height of the rocket after 2 seconds and after 4 seconds. What do you notice? **[2]**

$h(2) = -20 + 40 + 25 = 45$ m; $h(4) = -80 + 80 + 25 = 25$ m 1

At 4 s the rocket is back at roof height: the path is symmetric about $t = 2$ 1

D3. Case Study · Vegetable Plot [4]

(i) Find the zeroes of $A(x)$. **[1]**

$2x + 3 = 0$ or $x - 1 = 0$: zeroes $-\dfrac{3}{2}$ and 1 1

(ii) Find the sum and the product of the zeroes of $A(x)$. **[1]**

Sum $= -\dfrac{1}{2}$; product $= -\dfrac{3}{2}$ 1

(iii) Find a quadratic polynomial whose zeroes are twice the zeroes of $A(x)$. **[2]**

New zeroes -3 and 2: sum -1, product -6 1

Polynomial: $x^2 + x - 6$ 1

OR

(iii) If α, β are the zeroes of $A(x)$, find $\alpha^2 + \beta^2$ using the relationship with the coefficients. **[2]**

$\alpha + \beta = -\dfrac{1}{2}$, $\alpha\beta = -\dfrac{3}{2}$ 1

$\alpha^2 + \beta^2 = \dfrac{1}{4} + 3 = \dfrac{13}{4}$ 1

D4. Case Study · Bridge Arch [4]

(i) Find the zeroes of $p(x)$. How wide is the arch at the path? **[1]**

$\dfrac{1}{4}x^2 = 4 \Rightarrow x = \pm 4$; width $= 8$ m 1

(ii) Write the sum and product of the zeroes. Why is the sum zero? **[1]**

Sum $= 0$, product $= -16$. The sum is 0 because $b = 0$: the zeroes are equal and opposite 1

(iii) A new arch meets the path at $x = 0$ and $x = 8$ and has the same leading coefficient $-\frac{1}{4}$. Write its polynomial. **[2]**

$-\frac{1}{4}(x-0)(x-8)$ — 1

$= -\frac{1}{4}x^2 + 2x$ — 1

OR

(iii) For what value of k does $q(x) = -\frac{1}{4}x^2 + k$ have zeroes ± 6? What is the height of that arch? **[2]**

$-\frac{1}{4}(36) + k = 0 \Rightarrow k = 9$ — 1

Height $= q(0) = 9$ m — 1

PART E · SOLUTIONS

E Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. If α, β are the zeroes of $x^2 - px + q$, find $\alpha^3 + \beta^3$ in terms of p and q. Hence find it for $x^2 - 3x + 1$. **[3]**

$\alpha + \beta = p, \alpha\beta = q$ — ½

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = p^3 - 3pq$ — 1½

For $x^2 - 3x + 1$: $p = 3,\ q = 1 \rightarrow 27 - 9 = 18$ — 1

E2. If α, β are the zeroes of $2x^2 - 3x - 6$, find a quadratic polynomial whose zeroes are α^2 and β^2. **[3]**

$\alpha + \beta = \frac{3}{2}, \alpha\beta = -3$ — ½

Sum $= \alpha^2 + \beta^2 = \frac{9}{4} + 6 = \frac{33}{4}$ — 1

Product $= (\alpha\beta)^2 = 9$ — ½

$x^2 - \frac{33}{4}x + 9$, i.e. $4x^2 - 33x + 36$ — 1

E3. If α, β are the zeroes of $x^2 + px + 45$ and $(\alpha - \beta)^2 = 144$, find p. **[3]**

$\alpha + \beta = -p, \alpha\beta = 45$ — ½

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = p^2 - 180$ — 1

$p^2 - 180 = 144 \Rightarrow p^2 = 324$ — 1

$p = \pm 18$ — ½

E4. If α, β are the zeroes of $x^2 - 6x + a$ and $3\alpha + 2\beta = 20$, find a. **[3]**

$\alpha + \beta = 6$, so $3\alpha + 2\beta = 2(\alpha+\beta) + \alpha = 12 + \alpha$ — 1

$12 + \alpha = 20 \Rightarrow \alpha = 8, \beta = -2$ — 1

$a = \alpha\beta = -16$ — 1

E5. If α, β are the zeroes of $x^2 - 2x - 15$, form a quadratic polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$. **[3]**

$\alpha + \beta = 2,\ \alpha\beta = -15$	½
Sum $= 2(\alpha + \beta) + 2 = 6$	1
Product $= 4\alpha\beta + 2(\alpha + \beta) + 1 = -60 + 4 + 1 = -55$	1
Polynomial: $x^2 - 6x - 55$	½

E6. Show that $p(x) = x^2 + 4x + 7$ has no zero. What does this tell you about its graph? **[2]**

$x^2 + 4x + 7 = (x + 2)^2 + 3 \geq 3$ for every x	1
So $p(x)$ is never 0: no zeroes. Its graph lies wholly above the x-axis	1