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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 8

Introduction to Trigonometry

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check a value by putting in a known angle: for example, put $\theta = 30^\circ$ in an identity and see that both sides match.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN INTRODUCTION TO TRIGONOMETRY ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Evaluate (2 marks):** correct values put in 1 · simplified answer 1.
3. **One ratio given (2–3 marks):** third side 1 · the ratios needed 1 · final value 1.
4. **Prove an identity (3 marks):** a correct first step (common denominator, conjugate, or change to sin, cos) 1 · use of an identity 1 · reaching the other side 1.
5. **Find A and B (2 marks):** the two angle equations 1 · both angles 1.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Trigonometric ratios

A1. In $\triangle ABC$, right-angled at B, $AB = 9$ cm and $BC = 40$ cm. Find: **(i)** $\sin A$, $\cos A$ **(ii)** $\sin C$, $\cos C$ **[2]**

$AC = \sqrt{81 + 1600} = 41$ cm — ½

(i) $\sin A = \dfrac{40}{41}$, $\cos A = \dfrac{9}{41}$ — ½

(ii) $\sin C = \dfrac{9}{41}$, $\cos C = \dfrac{40}{41}$ — 1

A2. In the figure, $\angle Q = 90^\circ$, $PQ = 8$ cm and $PR = 17$ cm. Find $\tan P - \cot R$. **[2]**

$QR = \sqrt{289 - 64} = 15$ cm — ½

$\tan P = \dfrac{QR}{PQ} = \dfrac{15}{8}$, $\cot R = \dfrac{QR}{PQ} = \dfrac{15}{8}$ — 1

$\tan P - \cot R = 0$ — ½

A3. If $\sin A = \dfrac{5}{13}$, find $\cos A$ and $\tan A$. **[2]**

Opposite $5k$, hypotenuse $13k$, adjacent $= \sqrt{169k^2 - 25k^2} = 12k$ — 1

$\cos A = \dfrac{12}{13}$, $\tan A = \dfrac{5}{12}$ — 1

A4. Given $7 \cot A = 24$, find $\sin A$ and $\sec A$. **[2]**

$\cot A = \dfrac{24}{7}$: adjacent $24k$, opposite $7k$, hypotenuse $25k$ — 1

$\sin A = \dfrac{7}{25}$, $\sec A = \dfrac{25}{24}$ — 1

A5. Given $\sec\theta = \frac{25}{7}$, find all the other trigonometric ratios. [3]

$\cos\theta = \frac{7}{25}$: adjacent $7k$, hypotenuse $25k$, opposite $= \sqrt{625k^2 - 49k^2} = 24k$ — 1

$\sin\theta = \frac{24}{25}$, $\tan\theta = \frac{24}{7}$ — 1

$\operatorname{cosec}\theta = \frac{25}{24}$, $\cot\theta = \frac{7}{24}$ — 1

A6. If $\tan\theta = \frac{3}{4}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. [2]

$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$ — 1

$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ — 1

Q	Answer	Working / reason
A7	**(a)** $\sqrt{3}$	$A = 30^\circ$, so $\cot A = \sqrt{3}$.
A8	**(b)** $\sec A = \frac{12}{5}$ is possible	$\sec A = \frac{12}{5} > 1$ is possible (hypotenuse 12, adjacent 5). $\tan 60^\circ > 1$, and $\sin A$ cannot exceed 1.

Values at 0°, 30°, 45°, 60°, 90°

A9. Evaluate: **(i)** $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$ **(ii)** $4\cos^2 60^\circ + 3\sec^2 30^\circ - \tan^2 45^\circ$ [2]

(i) $\frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = 0$ — 1

(ii) $4\cdot\frac{1}{4} + 3\cdot\frac{4}{3} - 1 = 1 + 4 - 1 = 4$ — 1

A10. Evaluate $\frac{2\sin^2 45^\circ + 3\cot^2 60^\circ}{\tan^2 45^\circ + \cos^2 30^\circ}$. [2]

Numerator $= 2\cdot\frac{1}{2} + 3\cdot\frac{1}{3} = 2$; denominator $= 1 + \frac{3}{4} = \frac{7}{4}$ — 1

Value $= 2 \times \frac{4}{7} = \frac{8}{7}$ — 1

Q	Answer	Working / reason
A11	**(a)** $\cos 60°$	$\frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2} = \cos 60^\circ$.

A12. If $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$, with $A > B$ and $A + B \le 90^\circ$, find A and B. [2]

$A + B = 90^\circ$ and $A - B = 30^\circ$ — 1

$A = 60^\circ$, $B = 30^\circ$ — 1

A13. Show that $\sin 2A = 2\sin A\cos A$ when $A = 30^\circ$. [2]

LHS $= \sin 60^\circ = \frac{\sqrt{3}}{2}$ — 1

RHS $= 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} =$ LHS — 1

Identities

A14. Write $\sin A$ and $\tan A$ in terms of $\sec A$. [2]

$\tan^2 A = \sec^2 A - 1 \rightarrow \tan A = \sqrt{\sec^2 A - 1}$ 1

$\sin A = \tan A \cos A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$ 1

Q	Answer	Working / reason
A15	**(b)** 5	$5(\operatorname{cosec}^2\theta - \cot^2\theta) = 5 \times 1 = 5.$
A16	**(d)** 1	$\cos^2 A \cdot \sec^2 A = 1.$

A17. Prove that $(1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta) = 1.$ [2]

$\text{LHS} = \operatorname{cosec}^2\theta\,(1 - \cos^2\theta) = \operatorname{cosec}^2\theta \sin^2\theta$ 1

$= \dfrac{1}{\sin^2\theta} \cdot \sin^2\theta = 1 = \text{RHS}$ 1

A18. Prove that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta.$ [2]

$\text{LHS} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ 1

$= \dfrac{1}{\sin\theta\cos\theta} = \sec\theta\operatorname{cosec}\theta = \text{RHS}$ 1

A19. Prove that $\dfrac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2.$ [3]

$\text{RHS} = \left(\dfrac{1}{\cos A} - \dfrac{\sin A}{\cos A}\right)^2 = \dfrac{(1 - \sin A)^2}{\cos^2 A}$ 1

$= \dfrac{(1 - \sin A)^2}{1 - \sin^2 A} = \dfrac{(1 - \sin A)^2}{(1 - \sin A)(1 + \sin A)}$ 1

$= \dfrac{1 - \sin A}{1 + \sin A} = \text{LHS}$ 1

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	(c) $\frac{15}{8}$	Opposite $= \sqrt{289-64} = 15$, so $\tan A = \frac{15}{8}$.
B2	(a) $\frac{3}{\sqrt{5}}$	$\cos\theta = \frac{\sqrt{9-4}}{3} = \frac{\sqrt{5}}{3}$, so $\sec\theta = \frac{3}{\sqrt{5}}$.
B3	(b) $\sqrt{2}$	$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.
B4	(c) $\frac{1}{3}$	$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$, so it is $\frac{1}{3}$.
B5	(a) $30°$	$\sin 2\theta = \frac{\sqrt{3}}{2} \to 2\theta = 60^\circ \to \theta = 30^\circ$.
B6	(d) 2	$4\left(\frac{1}{16}+\frac{1}{16}\right) - 3\left(\frac{1}{2}-1\right) = \frac{1}{2}+\frac{3}{2} = 2$.
B7	(b) 9	$9(\sec^2\theta - \tan^2\theta) = 9$.
B8	(a)	$\sin\theta = \frac{\text{opp}}{\text{hyp}} < 1$. Both true; R explains A.

Short answer with reasons

B9. If $\sqrt{3}\tan\theta = 3$, find the value of $2\sin^2\theta - \cos^2\theta$. [2]

$\tan\theta = \sqrt{3} \to \theta = 60^\circ$ — 1

$2\cdot\frac{3}{4} - \frac{1}{4} = \frac{5}{4}$ — 1

B10. If $5\sin\theta = 3$, find $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$. [2]

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, so $\sec\theta = \frac{5}{4}$, $\tan\theta = \frac{3}{4}$ — 1

$\frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}} = \frac{1/2}{2} = \frac{1}{4}$ — 1

B11. Prove that $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$. [2]

$\text{RHS} = \frac{\sin A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A} = \frac{\sin A(1+\cos A)}{1-\cos^2 A}$ — 1

$= \frac{\sin A(1+\cos A)}{\sin^2 A} = \frac{1+\cos A}{\sin A} = \text{LHS}$ — 1

B12. Prove that $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$. [2]

$\text{LHS} = (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) = \sin^2\theta - \cos^2\theta$ — 1

$= (1 - \cos^2\theta) - \cos^2\theta = 1 - 2\cos^2\theta = \text{RHS}$ — 1

Long answer

B13. In $\triangle PQR$, right-angled at Q, $PQ = 3$ cm and $PR = 6$ cm. Find $\angle QPR$, $\angle PRQ$ and QR. [3]

$\cos P = \dfrac{PQ}{PR} = \dfrac{1}{2} \to \angle QPR = 60^\circ$ — 1

$\angle PRQ = 180^\circ - 90^\circ - 60^\circ = 30^\circ$ — 1

$QR = \sqrt{36 - 9} = 3\sqrt{3}$ cm — 1

B14. If $A = 60^\circ$ and $B = 30^\circ$, verify that (i) $\cos(A - B) = \cos A \cos B + \sin A \sin B$ and (ii) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$. [4]

(i) LHS $= \cos 30^\circ = \dfrac{\sqrt{3}}{2}$ — 1

RHS $= \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}$ ✓ — 1

(ii) LHS $= \tan 30^\circ = \dfrac{1}{\sqrt{3}}$ — 1

RHS $= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + 1} = \dfrac{2/\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}$ ✓ — 1

B15. Prove that $\dfrac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$. [3]

Write $1 = \sec^2\theta - \tan^2\theta$ in the numerator: $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta)$ — 1

$= (\sec\theta - \tan\theta)(1 + \sec\theta + \tan\theta)$, so LHS $= \sec\theta - \tan\theta$ — 1

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} = \dfrac{1 - \sin\theta}{\cos\theta} =$ RHS — 1

C PART C · SOLUTIONS
Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Standard values: evaluate and find angles

Q	Answer	Working / reason
C1	**(a)** $\dfrac{3}{2}$	$A = 60^\circ$: $\dfrac{3}{4} + 3 \cdot \dfrac{1}{4} = \dfrac{3}{2}$.
C2	**(c)** 60°	$\cos\theta = \dfrac{1}{2} \to \theta = 60^\circ$.
C3	**(d)** $\dfrac{1}{2}$	$\alpha + \beta = 90^\circ$, so $\sin 30^\circ = \dfrac{1}{2}$.
C4	**(c)** $\tan 60^\circ = 2\tan 30^\circ$	$2\tan 30^\circ = \dfrac{2}{\sqrt{3}} \neq \sqrt{3}$.
C5	**(d)**	$\sec\theta = \dfrac{1}{\cos\theta} \geq 1$, so A is false. R is true.

C6. Evaluate $\dfrac{3\tan^2 30^\circ + \sec^2 45^\circ}{\operatorname{cosec}^2 30^\circ - \cot^2 45^\circ}$. [2]

Numerator $= 3 \cdot \dfrac{1}{3} + 2 = 3$; denominator $= 4 - 1 = 3$ — 1

Value $= 1$ — 1

C7. If $x \sin^2 30^\circ + 2\cos^2 45^\circ = \tan^2 60^\circ$, find x. [2]

$\frac{x}{4} + 2 \cdot \frac{1}{2} = 3$ 1

$\frac{x}{4} = 2 \rightarrow x = 8$ 1

C8. If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$, with $A > B$ and $A + B \leq 90^\circ$, find A and B. [2]

$A - B = 30^\circ$ and $A + B = 60^\circ$ 1

$A = 45^\circ, B = 15^\circ$ 1

One ratio given: other ratios and expressions

Q	Answer	Working / reason
C9	**(b)** $\frac{p}{\sqrt{p^2+q^2}}$	Opposite p, adjacent q, hypotenuse $\sqrt{p^2+q^2}$.

C10. If $\sec\theta = \frac{13}{5}$, find $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$. [2]

$\cos\theta = \frac{5}{13}, \sin\theta = \frac{12}{13}$ 1

$\frac{24 - 15}{48 - 45} = \frac{9}{3} = 3$ 1

C11. If $\operatorname{cosec}\theta = \frac{17}{8}$, find $\frac{1 - \tan^2\theta}{1 + \tan^2\theta}$. [2]

$\sin\theta = \frac{8}{17}, \cos\theta = \frac{15}{17}, \tan\theta = \frac{8}{15}$ 1

$\frac{1 - \frac{64}{225}}{1 + \frac{64}{225}} = \frac{161}{289}$ 1

C12. In $\triangle PQR$, right-angled at Q, $PR = 41$ cm and $QR = 9$ cm. If $\angle QPR = \theta$, find (i) $1 + \cot^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$. [3]

$PQ = \sqrt{1681 - 81} = 40$ cm; $\sin\theta = \frac{9}{41}, \cos\theta = \frac{40}{41}$ 1

(i) $1 + \cot^2\theta = \operatorname{cosec}^2\theta = \frac{1681}{81}$ 1

(ii) $\frac{1600 - 81}{1681} = \frac{1519}{1681}$ 1

Identities: simplify and prove

Q	Answer	Working / reason
C13	**(c)** -1	$\cot^2\theta - \operatorname{cosec}^2\theta = -1$.
C14	**(b)** 1	$\operatorname{cosec}^2 A \cdot \sin^2 A = 1$.

C15. Prove that $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}=\sin A+\cos A$. **[3]**

$\dfrac{\cos A}{1-\frac{\sin A}{\cos A}}=\dfrac{\cos^2 A}{\cos A-\sin A}$ and $\dfrac{\sin A}{1-\frac{\cos A}{\sin A}}=\dfrac{\sin^2 A}{\sin A-\cos A}$ — 1

$\text{LHS}=\dfrac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$ — 1

$=\dfrac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}=\sin A+\cos A=\text{RHS}$ — 1

C16. Prove that $\sec A(1-\sin A)(\sec A+\tan A)=1$. **[2]**

$\text{LHS}=\dfrac{1}{\cos A}(1-\sin A)\cdot\dfrac{1+\sin A}{\cos A}=\dfrac{1-\sin^2 A}{\cos^2 A}$ — 1

$=\dfrac{\cos^2 A}{\cos^2 A}=1=\text{RHS}$ — 1

C17. If $x=2\sec\theta$ and $y=3\tan\theta$, prove that $9x^2-4y^2=36$. **[2]**

$\sec\theta=\dfrac{x}{2}$, $\tan\theta=\dfrac{y}{3}$; $\sec^2\theta-\tan^2\theta=1\rightarrow\dfrac{x^2}{4}-\dfrac{y^2}{9}=1$ — 1

Multiply by 36: $9x^2-4y^2=36$ — 1

C18. A student simplified $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}$ like this. Find the wrong step and complete the working correctly. **[3]**

Step 1: $=\dfrac{(1+\cos\theta)^2+\sin^2\theta}{\sin\theta(1+\cos\theta)}$ Step 2: $=\dfrac{1+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$ Step 3: $=\dfrac{2}{\sin\theta(1+\cos\theta)}$

Step 2 is wrong: $(1+\cos\theta)^2=1+2\cos\theta+\cos^2\theta$ — 1

Correct: $\dfrac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}=\dfrac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ — 1

$=\dfrac{2}{\sin\theta}=2\,\text{cosec}\,\theta$ — 1

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Wheelchair Ramp [4]

(i) Find the horizontal distance AC. **[1]**

$AC = \sqrt{25 - 1.96} = \sqrt{23.04} = 4.8$ m — 1

(ii) Find $\tan\theta$. **[1]**

$\tan\theta = \dfrac{1.4}{4.8} = \dfrac{7}{24}$ — 1

(iii) Find $\operatorname{cosec}\theta + \cot\theta$. **[2]**

$\operatorname{cosec}\theta = \dfrac{5}{1.4} = \dfrac{25}{7}$, $\cot\theta = \dfrac{24}{7}$ — 1

$\text{Sum} = \dfrac{49}{7} = 7$ — 1

OR

(iii) A safety rule says $\tan\theta$ must not exceed $\dfrac{1}{12}$. Does this ramp meet the rule? For the same rise, how long must the horizontal distance be to give $\tan\theta = \dfrac{1}{12}$? **[2]**

$\dfrac{7}{24} \approx 0.29 > \dfrac{1}{12} \approx 0.083$: it does **not** meet the rule — 1

$\dfrac{1.4}{AC} = \dfrac{1}{12} \rightarrow AC = 16.8$ m — 1

D2. Case Study · Solar Panel [4]

(i) Find θ. **[1]**

$\sin\theta = \dfrac{1}{2} \rightarrow \theta = 30^\circ$ — 1

(ii) Find the horizontal distance covered by the panel. **[1]**

$2\cos 30^\circ = \sqrt{3} \approx 1.73$ m — 1

(iii) In winter the tilt is changed to 45°. How high is the upper edge now? **[2]**

$\text{Height} = 2\sin 45^\circ$ — 1

$= \dfrac{2}{\sqrt{2}} = \sqrt{2} \approx 1.41$ m — 1

OR

(iii) For what tilt is the upper edge $\sqrt{3}$ m above the ground? What horizontal distance does the panel then cover? **[2]**

$2\sin\theta = \sqrt{3} \rightarrow \theta = 60^\circ$ — 1

$\text{Horizontal distance} = 2\cos 60^\circ = 1$ m — 1

D3. Case Study · Cycle Track [4]

(i) Find $\sin\theta$ and $\cos\theta$. **[1]**

Hypotenuse $13k$: $\sin\theta = \dfrac{5}{13}$, $\cos\theta = \dfrac{12}{13}$ — 1

(ii) A cyclist rides 26 m along the track. How much height does she gain? **[1]**

$\text{Height} = 26\sin\theta = 26 \times \frac{5}{13} = 10$ m — 1

(iii) Using your values, show that $\sec^2\theta - \tan^2\theta = 1$. **[2]**

$\sec^2\theta = \frac{169}{144}$, $\tan^2\theta = \frac{25}{144}$ — 1

$\text{Difference} = \frac{144}{144} = 1$ ✓ — 1

OR

(iii) Find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$. **[2]**

$\frac{\frac{5}{13} + \frac{12}{13}}{\frac{5}{13} - \frac{12}{13}} = \frac{17}{-7}$ — 1

$= -\frac{17}{7}$ — 1

D4. Case Study · Step-Ladder [4]

(i) What angle does each leg make with the ground? **[1]**

$\triangle TPQ$ is equilateral (all sides 2 m), so the angle is $60°$ — 1

(ii) How high is the top of the ladder? **[1]**

$\text{Height} = 2\sin 60^\circ = \sqrt{3} \approx 1.73$ m — 1

(iii) The feet are moved apart until the top is $\sqrt{2}$ m high. Find the angle each leg now makes with the ground, and the distance PQ. **[2]**

$2\sin\theta = \sqrt{2} \to \sin\theta = \frac{1}{\sqrt{2}} \to \theta = 45^\circ$ — 1

$PQ = 2 \times 2\cos 45^\circ = 2\sqrt{2} \approx 2.83$ m — 1

OR

(iii) If each leg makes $30°$ with the ground, find the height of the top and the distance between the feet. **[2]**

$\text{Height} = 2\sin 30^\circ = 1$ m — 1

$PQ = 2 \times 2\cos 30^\circ = 2\sqrt{3} \approx 3.46$ m — 1

E PART E · SOLUTIONS
Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. Prove that $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$. [3]

Write $1 = \operatorname{cosec}^2\theta - \cot^2\theta$ in the numerator: $(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta)$ — 1

$= (\operatorname{cosec}\theta + \cot\theta)(1 - \operatorname{cosec}\theta + \cot\theta)$, so LHS $= \operatorname{cosec}\theta + \cot\theta$ — 1

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1 + \cos\theta}{\sin\theta} = \text{RHS}$ — 1

E2. If $\operatorname{cosec}\theta + \cot\theta = q$, show that $\cos\theta = \dfrac{q^2-1}{q^2+1}$. **[3]**

$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$, so $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{q}$ — 1

Adding and subtracting: $\operatorname{cosec}\theta = \dfrac{q^2+1}{2q}$, $\cot\theta = \dfrac{q^2-1}{2q}$ — 1

$\cos\theta = \dfrac{\cot\theta}{\operatorname{cosec}\theta} = \dfrac{q^2-1}{q^2+1}$ — 1

E3. If $3\sin\theta + 4\cos\theta = 5$, find $\tan\theta$. **[3]**

$(3\sin\theta + 4\cos\theta)^2 + (4\sin\theta - 3\cos\theta)^2 = 25(\sin^2\theta + \cos^2\theta) = 25$ — 1

So $(4\sin\theta - 3\cos\theta)^2 = 25 - 25 = 0 \rightarrow 4\sin\theta = 3\cos\theta$ — 1

$\tan\theta = \dfrac{3}{4}$ — 1

E4. Find the acute angle θ if $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. **[3]**

Divide by $\cos\theta$: $\dfrac{1+\tan\theta}{1-\tan\theta} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ — 1

Cross-multiply: $2\sqrt{3}\tan\theta = 2 \rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$ — 1

$\theta = 30^\circ$ — 1

E5. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$. **[3]**

Let $a = \sin^2\theta$, $b = \cos^2\theta$, so $a + b = 1$ — 1

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ — 1

$= 1 - 3\sin^2\theta\cos^2\theta = \text{RHS}$ — 1

E6. Find θ ($0^\circ < \theta < 90^\circ$) if $2\sin^2\theta - 3\sin\theta + 1 = 0$. **[3]**

$(2\sin\theta - 1)(\sin\theta - 1) = 0$ — 1

$\sin\theta = \dfrac{1}{2}$ or $\sin\theta = 1$; $\sin\theta = 1$ gives $\theta = 90^\circ$, which is not allowed — 1

$\theta = 30^\circ$ — 1