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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 12

Surface Areas and Volumes

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Before computing, list the parts of the solid and the faces that are hidden or added.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN SURFACE AREAS AND VOLUMES ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **3-mark joined solid:** each part's area or volume 1 each · total with unit 1.
3. **5-mark question:** slant height or missing length 1 · each area or volume 1 · total 1 · cost or final answer 1.
4. Write the formula for each part before putting in the numbers.
5. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Surface area of joined solids

A1. Two cubes, each of volume 216 cm³, are joined end to end. Find the surface area of the cuboid formed. [2]

Edge $= \sqrt[3]{216} = 6$ cm; the cuboid is $12 \times 6 \times 6$ cm — 1

Surface area $= 2(12 \times 6 + 6 \times 6 + 6 \times 12) = 2 \times 180 = 360$ cm² — 1

A2. A vessel is a hollow hemisphere with a hollow cylinder on top. The diameter is 14 cm and the total height is 15 cm. Find the inner surface area of the vessel. [3]

$r = 7$ cm; height of the cylinder $= 15 - 7 = 8$ cm — 1

Hemisphere: $2\pi r^2 = 2 \times \frac{22}{7} \times 49 = 308$ cm²; cylinder: $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 8 = 352$ cm² — 1

Inner surface area $= 308 + 352 = 660$ cm² — 1

A3. A toy is a cone of radius 6 cm on a hemisphere of the same radius. The total height of the toy is 14 cm. Find its total surface area. (Use $\pi = 3.14$.) [3]

Height of cone $= 14 - 6 = 8$ cm; slant height $l = \sqrt{36 + 64} = 10$ cm — 1

Hemisphere: $2 \times 3.14 \times 36 = 226.08$ cm²; cone: $3.14 \times 6 \times 10 = 188.4$ cm² — 1

Total surface area $= 226.08 + 188.4 = 414.48$ cm² — 1

A4. A cube of edge 12 cm has a hemisphere on one face. Find the greatest possible diameter of the hemisphere and, for it, the surface area of the solid. (Use $\pi = 3.14$.) [2]

Greatest diameter $=$ edge $= 12$ cm, so $r = 6$ cm; area $= 6a^2 - \pi r^2 + 2\pi r^2$ — 1

$= 6a^2 + \pi r^2 = 864 + 113.04 = 977.04$ cm² — 1

A5. A hemisphere of diameter 10 cm is scooped out of one face of a cube of edge 10 cm. Find the surface area of the solid left. (Use $\pi = 3.14$.) [2]

Surface area $= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$ with $a = 10$, $r = 5$ 1

$= 600 + 3.14 \times 25 = 600 + 78.5 = 678.5$ cm² 1

A6. A capsule is a cylinder with two hemispheres stuck to its ends. Its total length is 20 mm and its diameter is 6 mm. Find its surface area. [3]

$r = 3$ mm; length of the cylinder $= 20 - 6 = 14$ mm 1

Cylinder: $2 \times \frac{22}{7} \times 3 \times 14 = 264$ mm²; two hemispheres: $4 \times \frac{22}{7} \times 9 \approx 113.14$ mm² 1

Surface area $= 264 + 113.14 = 377.14$ mm² 1

A7. A tent is a cylinder of diameter 14 m and height 3 m with a cone on top of slant height 10 m. Find the area of canvas used and its cost at ₹500 per m². [3]

Cylinder: $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 3 = 132$ m² 1

Cone: $\pi rl = \frac{22}{7} \times 7 \times 10 = 220$ m², so canvas $= 352$ m² 1

Cost $= 352 \times 500 =$ ₹1,76,000 1

A8. A cone of the same height and diameter is scooped out of a solid cylinder of height 4 cm and diameter 6 cm. Find the total surface area of the solid left. [3]

$r = 3$ cm, $h = 4$ cm, so the slant height of the cone is $l = 5$ cm 1

TSA $= 2\pi rh + \pi r^2 + \pi rl = \pi(24 + 9 + 15) = 48\pi$ 1

$= 48 \times \frac{22}{7} \approx 150.86$ cm² 1

A9. A wooden article is a cylinder of height 12 cm and radius 3.5 cm with a hemisphere of the same radius scooped out of each end. Find its total surface area. [3]

Curved surface of cylinder $= 2 \times \frac{22}{7} \times 3.5 \times 12 = 264$ cm² 1

Two hemispherical hollows $= 2 \times 2\pi r^2 = 4 \times \frac{22}{7} \times 12.25 = 154$ cm² 1

Total surface area $= 264 + 154 = 418$ cm² 1

Volume of joined solids

A10. A solid is a cone of radius 3 cm and height 3 cm on a hemisphere of the same radius. Find its volume in terms of π. [2]

Hemisphere $= \frac{2}{3}\pi \times 27 = 18\pi$; cone $= \frac{1}{3}\pi \times 9 \times 3 = 9\pi$ 1

Volume $= 27\pi$ cm³ 1

A11. A model is a cylinder with a cone at each end. The diameter is 4 cm, the total length is 14 cm and each cone is 3 cm long. Find the volume of the model. [3]

$r = 2$ cm; length of the cylinder $= 14 - 6 = 8$ cm 1

Cylinder $= \pi \times 4 \times 8 = 32\pi$; two cones $= 2 \times \frac{1}{3}\pi \times 4 \times 3 = 8\pi$ 1

Volume $= 40\pi = 40 \times \frac{22}{7} \approx 125.71$ cm³ 1

A12. Each gulab jamun is a cylinder with two hemispherical ends, 5.6 cm long and 2.8 cm in diameter. It holds syrup equal to 30% of its volume. Find the syrup in 50 gulab jamuns. [3]

$r = 1.4$, $h = 2.8$ cm; one $= \pi r^2\left(h + \frac{4}{3}r\right) = \frac{22}{7} \times 1.96 \times \frac{14}{3} \approx 28.75$ cm³ 1

50 gulab jamuns $= 50 \times 28.75 \approx 1437.33$ cm³ 1

Syrup $= 30\%$ of $1437.33 = 431.2$ cm³ 1

A13. A pen stand is a wooden cuboid $20 \times 12 \times 4$ cm with four conical holes, each of radius 0.7 cm and depth 2.1 cm. Find the volume of wood in the stand. [3]

Cuboid $= 20 \times 12 \times 4 = 960$ cm³ 1

One cone $= \frac{1}{3} \times \frac{22}{7} \times 0.49 \times 2.1 = 1.078$ cm³; four cones $= 4.312$ cm³ 1

Wood $= 960 - 4.312 \approx 955.69$ cm³ 1

A14. An inverted cone of radius 6 cm and height 10 cm is full of water. Lead shots, each a sphere of radius 0.5 cm, are dropped in until one-fifth of the water flows out. How many shots are dropped? [3]

Water out $= \frac{1}{5} \times \frac{1}{3}\pi \times 36 \times 10 = 24\pi$ cm³ 1

One shot $= \frac{4}{3}\pi \times 0.125 = \frac{\pi}{6}$ cm³ 1

Number of shots $= 24\pi \div \frac{\pi}{6} = 144$ 1

A15. An iron pole is a cylinder of height 200 cm and base diameter 28 cm, with a cylinder of height 70 cm and radius 7 cm on top. Find its mass if 1 cm³ of iron has mass 8 g. [3]

Lower part $= \frac{22}{7} \times 196 \times 200 = 123200$ cm³; upper part $= \frac{22}{7} \times 49 \times 70 = 10780$ cm³ 1

Volume $= 133980$ cm³ 1

Mass $= 133980 \times 8 = 1071840$ g $= 1071.84$ kg 1

A16. A solid is a cone of radius 30 cm and height 60 cm on a hemisphere of radius 30 cm. It stands upright in a cylinder of radius 30 cm and height 90 cm that was full of water, touching the bottom. Find the water left in the cylinder in m³. (Use $\pi = 3.14$.) [4]

Cylinder $= 3.14 \times 900 \times 90 = 254340$ cm³ 1

Cone $= \frac{1}{3} \times 3.14 \times 900 \times 60 = 56520$ cm³ 1

Hemisphere $= \frac{2}{3} \times 3.14 \times 27000 = 56520$ cm³ 1

Water left $= 254340 - 113040 = 141300$ cm³ $= 0.1413$ m³ 1

A17. A glass vessel is a sphere of diameter 14 cm with a cylindrical neck 7 cm long and 2 cm in diameter. Find the volume of water it can hold. [2]

Sphere $= \frac{4}{3} \times \frac{22}{7} \times 343 \approx 1437.33$ cm³; neck $= \frac{22}{7} \times 1 \times 7 = 22$ cm³ — 1

Volume $= 1437.33 + 22 = 1459.33$ cm³ — 1

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(c)** a cone and a cylinder	The sharpened tip is a cone and the rest is a cylinder.
B2	**(b)** 89.83 cm³	$r = 3.5$, $h = 7$: $\frac{1}{3} \times \frac{22}{7} \times 12.25 \times 7 \approx 89.83$ cm³.
B3	**(d)** 462 cm²	$3\pi r^2 = 3 \times 154 = 462$ cm².
B4	**(a)** 288 cm²	Each is $6 \times 6 \times 3$: $2(36 + 18 + 18) = 144$ cm²; two give 288 cm².
B5	**(c)** 3 : 1 : 2	$\pi r^3 : \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 3 : 1 : 2$.
B6	**(a)**	Both are true and R gives the reason for A.
B7	**(d)**	The cuboid $8 \times 4 \times 4$ has area $2(32 + 16 + 32) = 160$ cm², so A is false. R is true.
B8	**(b)**	Both are true, but the area formula does not explain the volume formula.

Short answer with reasons

B9. A solid is a cylinder of radius 7 cm and height 10 cm with a cone of the same radius and height 24 cm on top. Find its volume. [2]

Cylinder $= \frac{22}{7} \times 49 \times 10 = 1540$ cm³; cone $= \frac{1}{3} \times \frac{22}{7} \times 49 \times 24 = 1232$ cm³ — 1

Volume $= 1540 + 1232 = 2772$ cm³ — 1

B10. A cylindrical hole of radius 1 cm is drilled straight through a cuboid $8 \times 6 \times 4$ cm along its 4 cm height. Find the volume left. [2]

Cuboid $= 192$ cm³; hole $= \frac{22}{7} \times 1 \times 4 \approx 12.57$ cm³ — 1

Volume left $= 192 - 12.57 = 179.43$ cm³ — 1

B11. A capsule is a cylinder with hemispherical ends. Its total length is 21 mm and its diameter is 7 mm. Find its volume. [3]

$r = 3.5$ mm; cylinder length $= 21 - 7 = 14$ mm — 1

Cylinder $= \frac{22}{7} \times 12.25 \times 14 = 539$ mm³; two hemispheres $= \frac{4}{3} \times \frac{22}{7} \times 42.875 \approx 179.67$ mm³ — 1

Volume $= 539 + 179.67 = 718.67$ mm³ — 1

B12. A hemisphere of diameter 14 cm stands on one face of a cube of edge 28 cm. Find the total surface area and the volume of the solid. [3]

$\text{TSA} = 6a^2 + \pi r^2 = 6 \times 784 + \frac{22}{7} \times 49 = 4704 + 154 = 4858 \text{ cm}^2$ 1

$\text{Hemisphere} = \frac{2}{3} \times \frac{22}{7} \times 343 \approx 718.67 \text{ cm}^3$ 1

$\text{Volume} = 28^3 + 718.67 = 21952 + 718.67 = 22670.67 \text{ cm}^3$ 1

Long answer

B13. A water tank is a cylinder of diameter 14 m and height 5 m with a hemispherical bottom. Find (i) its capacity in litres, (ii) the cost of painting the inside (no lid) at ₹20 per m². [5]

$\text{Cylinder} = \frac{22}{7} \times 49 \times 5 = 770 \text{ m}^3$ 1

$\text{Hemisphere} = \frac{2}{3} \times \frac{22}{7} \times 343 \approx 718.67 \text{ m}^3$ 1

(i) $\text{Capacity} = 1488.67 \text{ m}^3 \approx 14{,}88{,}667 \text{ litres}$ 1

$\text{Inside area} = 2\pi rh + 2\pi r^2 = 220 + 308 = 528 \text{ m}^2$ 1

(ii) $\text{Cost} = 528 \times 20 = ₹10{,}560$ 1

B14. A metal pipe is 21 cm long. Its inner diameter is 4 cm and outer diameter is 6 cm. Find (i) the volume of metal, (ii) its total surface area. [3]

(i) $\text{Volume} = \pi(R^2 - r^2)h = \frac{22}{7} \times (9 - 4) \times 21 = 330 \text{ cm}^3$ 1

$\text{Inner and outer curved surfaces} = 2\pi(R + r)h = 2 \times \frac{22}{7} \times 5 \times 21 = 660 \text{ cm}^2$ 1

(ii) Two rings $= 2\pi(R^2 - r^2) \approx 31.43 \text{ cm}^2$, so $\text{TSA} = 691.43 \text{ cm}^2$ 1

C

PART C · SOLUTIONS

Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Joined solids: one-mark questions

Q	Answer	Working / reason
C1	**(b)** 90 cm²	Edge 3 cm; cuboid $6 \times 3 \times 3$: $2(18 + 9 + 18) = 90$ cm².
C2	**(c)** 616 cm²	They form a sphere: $4\pi r^2 = 616$ cm².
C3	**(a)** $1437\frac{1}{3}$ cm³	$r = 7$: $\frac{4}{3} \times \frac{22}{7} \times 343 = 1437\frac{1}{3}$ cm³.
C4	**(d)** 13 cm	$l = \sqrt{25 + 144} = 13$ cm.
C5	**(a)** sphere	A sphere has no flat face, so its curved and total areas are the same.
C6	**(d)**	The hemispheres have $2 \times 3\pi r^2 = 6\pi r^2$, more than $4\pi r^2$, so A is false. R is true.

C7. A solid cylinder of radius 3.5 cm and height 10 cm has a hemisphere of the same radius on one end. Find its total surface area. [2]

Side $= 2 \times \frac{22}{7} \times 3.5 \times 10 = 220$; base $\pi r^2 = 38.5$; hemisphere $2\pi r^2 = 77$ (cm²) 1

TSA $= 220 + 38.5 + 77 = 335.5$ cm² 1

Joined and hollow solids: volume, surface area, cost

C8. A solid is a cylinder with a hemisphere at each end. Its total length is 19 cm and its diameter is 7 cm. Find its volume. [3]

$r = 3.5$ cm; cylinder length $= 19 - 7 = 12$ cm 1

Cylinder $= \frac{22}{7} \times 12.25 \times 12 = 462$ cm³; two hemispheres ≈ 179.67 cm³ 1

Volume $= 462 + 179.67 = 641.67$ cm³ 1

C9. A godown is a cylinder of radius 7 m with a hemispherical dome on top. It holds $1642\frac{2}{3}$ m³ of air. Find the height of the cylindrical part. [3]

Dome $= \frac{2}{3} \times \frac{22}{7} \times 343 = 718\frac{2}{3}$ m³ 1

Cylinder $= 1642\frac{2}{3} - 718\frac{2}{3} = 924$ m³ 1

$\frac{22}{7} \times 49 \times h = 924$, so $h = 6$ m 1

C10. A metal pipe 14 cm long contains 704 cm³ of metal. Its outer radius is 5 cm. Find its inner radius. [3]

$\pi(R^2 - r^2)h = 704$: $\frac{22}{7} \times (25 - r^2) \times 14 = 704$ 1

$44(25 - r^2) = 704$, so $25 - r^2 = 16$ 1

$r = 3$ cm 1

C11. A solid is a cone on a hemisphere, both of radius 6 cm. Its volume is 216π cm³. Find the height of the cone. [3]

Hemisphere $= \frac{2}{3}\pi \times 216 = 144\pi$ cm³ 1

Cone $= 216\pi - 144\pi = 72\pi = \frac{1}{3}\pi \times 36 \times h$ 1

$12h = 72$, so $h = 6$ cm 1

C12. A toy is a cone of radius 9 cm and height 12 cm on a hemisphere of the same radius. Find its total surface area and volume. (Use $\pi = 3.14$.) [5]

Slant height $l = \sqrt{81 + 144} = 15$ cm 1

Hemisphere: $2 \times 3.14 \times 81 = 508.68$ cm²; cone: $3.14 \times 9 \times 15 = 423.9$ cm² 1

TSA $= 508.68 + 423.9 = 932.58$ cm² 1

Hemisphere: $\frac{2}{3} \times 3.14 \times 729 = 1526.04$ cm³; cone: $\frac{1}{3} \times 3.14 \times 81 \times 12 = 1017.36$ cm³ 1

Volume $= 1526.04 + 1017.36 = 2543.4$ cm³ 1

C13. A tent is a cylinder of radius 7 m and height 4 m with a conical top of slant height 10 m. 10% extra canvas is needed for stitching. Find the canvas needed and its cost at ₹600 per m². **[5]**

Cylinder: $2 \times \frac{22}{7} \times 7 \times 4 = 176$ m² — 1

Cone: $\frac{22}{7} \times 7 \times 10 = 220$ m² — 1

Canvas on the tent $= 396$ m² — 1

With 10% extra: $396 \times 1.1 = 435.6$ m² — 1

Cost $= 435.6 \times 600 =$ ₹2,61,360 — 1

C14. A hemisphere of diameter 14 cm is scooped out of one face of a solid cube of edge 14 cm. Find the volume and the total surface area of the solid left. **[5]**

Cube $= 14^3 = 2744$ cm³ — 1

Hemisphere $= \frac{2}{3} \times \frac{22}{7} \times 343 \approx 718.67$ cm³ — 1

Volume left $= 2744 - 718.67 = 2025.33$ cm³ — 1

Surface: cube $6 \times 196 = 1176$ cm², minus the circle πr^2, plus the bowl $2\pi r^2$ — 1

TSA $= 1176 - 154 + 308 = 1330$ cm² — 1

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Ice-cream Cone [4]

(i) Find the slant height of the cone. **[1]**

$l = \sqrt{9 + 16} = 5$ cm — 1

(ii) Find the volume of the hemisphere of ice cream. **[1]**

$\frac{2}{3} \times 3.14 \times 27 = 56.52$ cm³ — 1

(iii) Find the total volume of ice cream. **[2]**

Cone $= \frac{1}{3} \times 3.14 \times 9 \times 4 = 37.68$ cm³ — 1

Total $= 37.68 + 56.52 = 94.2$ cm³ — 1

OR

(iii) Find the area of wafer needed for the curved surface of one cone, and for 20 cones. **[2]**

One cone: $\pi r l = 3.14 \times 3 \times 5 = 47.1$ cm² — 1

20 cones: 942 cm² — 1

D2. Case Study · Grain Silo [4]

(i) Find the slant height of the roof. **[1]**

$l = \sqrt{16 + 9} = 5$ m — 1

(ii) Find the volume of the cylindrical part. **[1]**

$3.14 \times 16 \times 9 = 452.16$ m³ 1

(iii) Find the total capacity of the silo. **[2]**

Roof $= \frac{1}{3} \times 3.14 \times 16 \times 3 = 50.24$ m³ 1

Capacity $= 452.16 + 50.24 = 502.4$ m³ 1

OR

(iii) Find the area of metal sheet needed for the curved wall and the roof (no floor). **[2]**

Wall $= 2 \times 3.14 \times 4 \times 9 = 226.08$ m²; roof $= 3.14 \times 4 \times 5 = 62.8$ m² 1

Sheet $= 288.88$ m² 1

D3. Case Study · Toy Rocket [4]

(i) Find the slant height of the cone. **[1]**

$l = \sqrt{25 + 144} = 13$ cm 1

(ii) Find the curved surface area of the cone. **[1]**

$\pi r l = 3.14 \times 5 \times 13 = 204.1$ cm² 1

(iii) The whole outside, including the base, is painted at ₹0.50 per cm². Find the area painted and the cost. **[2]**

Area $= 204.1 + 2 \times 3.14 \times 5 \times 20 + 3.14 \times 25 = 204.1 + 628 + 78.5 = 910.6$ cm² 1

Cost $= 910.6 \times 0.50 =$ ₹455.30 1

OR

(iii) Find the volume of wood in the rocket. **[2]**

Cylinder $= 3.14 \times 25 \times 20 = 1570$ cm³; cone $= \frac{1}{3} \times 3.14 \times 25 \times 12 = 314$ cm³ 1

Volume $= 1884$ cm³ 1

D4. Case Study · Bird Bath [4]

(i) Find the curved surface area of the cylinder. **[1]**

$2 \times \frac{22}{7} \times 35 \times 100 = 22000$ cm² 1

(ii) Find the surface area of the hollow. **[1]**

$2\pi r^2 = 2 \times \frac{22}{7} \times 1225 = 7700$ cm² 1

(iii) Find the total area to be painted (all except the base) in m². **[2]**

Area $= 22000 + 7700 = 29700$ cm² 1

$= 2.97$ m² 1

OR

(iii) Find the volume of material in the bird bath. **[2]**

Cylinder $= \frac{22}{7} \times 1225 \times 100 = 385000$ cm³; hollow $= \frac{2}{3} \times \frac{22}{7} \times 42875 \approx 89833.33$ cm³ 1

Material $= 295166.67$ cm³ ≈ 0.30 m³ 1

PART E · SOLUTIONS

E Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. A solid cone (radius 3 cm, height 4 cm) on a hemisphere of radius 3 cm is put fully under water in a cylinder of radius 10 cm. Find the rise in the water level. [3]

Volume of the solid $= \frac{2}{3}\pi \times 27 + \frac{1}{3}\pi \times 9 \times 4 = 18\pi + 12\pi = 30\pi$ cm³ 1

Rise h: $\pi \times 10^2 \times h = 30\pi$ 1

$h = 0.3$ cm 1

E2. A hemisphere of radius r is scooped out of each end of a solid cylinder of radius r and height $2r$. Show that the volume left is $\frac{2}{3}\pi r^3$ and the total surface area is $8\pi r^2$. [3]

Volume $= \pi r^2(2r) - 2 \times \frac{2}{3}\pi r^3 = 2\pi r^3 - \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$ 1

Curved surface of cylinder $= 2\pi r \times 2r = 4\pi r^2$ 1

Two hollows $= 2 \times 2\pi r^2 = 4\pi r^2$, so TSA $= 8\pi r^2$ 1

E3. Find the ratio of the volumes of the largest cone and largest sphere cut from a cube of edge a. [3]

Cone: $r = \frac{a}{2}$, $h = a$: $\frac{1}{3}\pi\frac{a^2}{4}a = \frac{\pi a^3}{12}$ 1

Sphere: $r = \frac{a}{2}$: $\frac{4}{3}\pi\frac{a^3}{8} = \frac{\pi a^3}{6}$ 1

Ratio $= \frac{1}{12} : \frac{1}{6} = 1 : 2$ 1

E4. A toy is a cone on a hemisphere of the same radius. The volumes of the cone and the hemisphere are equal, and the toy is 21 cm tall. Find its total surface area. (Use $\sqrt{5} = 2.236$.) [3]

$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$ gives $h = 2r$; total height $3r = 21$, so $r = 7$ cm, $h = 14$ cm 1

$l = \sqrt{49 + 196} = 7\sqrt{5}$ cm; cone $= \frac{22}{7} \times 7 \times 7\sqrt{5} = 154\sqrt{5} \approx 344.34$ cm² 1

Hemisphere $= 308$ cm², so TSA ≈ 652.34 cm² 1

E5. A conical hollow of diameter 12 cm and depth 8 cm is cut into one end of a solid cylinder of diameter 12 cm and height 15 cm. Find the total surface area of the solid left. [3]

$r = 6$ cm; slant height of the hollow $l = \sqrt{36 + 64} = 10$ cm 1

TSA $= 2\pi rh + \pi r^2 + \pi rl = \pi(180 + 36 + 60) = 276\pi$ 1

$= 276 \times \frac{22}{7} \approx 867.43$ cm² 1

E6. A sphere fits exactly inside a cylinder, touching its curved side, top and bottom. Show that the ratio of their volumes and the ratio of their total surface areas are both 2 : 3. [3]

Cylinder: radius r, height $2r$ 1

Volumes: $\frac{4}{3}\pi r^3 : \pi r^2(2r) = \frac{4}{3} : 2 = 2 : 3$ 1

Areas: $4\pi r^2 : (2\pi r \times 2r + 2\pi r^2) = 4 : 6 = 2 : 3$ 1