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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 1

Real Numbers

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

✓	**Marking Guide** How step marks are given	2
A	**NCERT Practice – Solutions** 13 solutions	2
B	**Exemplar Practice – Solutions** 18 solutions	4
C	**Board Pattern Practice – Solutions** 43 solutions	6
D	**Case-Based Questions – Solutions** 4 case studies, all parts and ORs	11
E	**Challenge Questions – Solutions** 7 solutions	12

HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check an HCF by dividing both numbers by it, and an LCM by dividing it by both numbers.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN REAL NUMBERS ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **$\sqrt{p}$ is irrational (3 marks):** assume $\frac{a}{b}$ with a, b coprime ½ · p divides a^2 so p divides a 1 · p divides b 1 · contradiction and conclusion ½.
3. **$a + b\sqrt{p}$ is irrational:** assume rational ½ · make $\sqrt{p}$ the subject 1 · say the right side is rational, so contradiction ½ (3-mark versions give 1 more for the reasoning).
4. **HCF and LCM:** prime factorisation 1 · HCF ½ · LCM ½; HCF × LCM = product for the missing one.
5. **Word problems:** deciding HCF or LCM (with the reason) · working · the final answer in the units or time asked.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Prime factorisation (Fundamental Theorem of Arithmetic)

A1. Express each number as a product of its prime factors: **(i)** 180 **(ii)** 252 **(iii)** 4725 **(iv)** 6435 **(v)** 4199 **[5]**

(i) $180 = 2^2 \times 3^2 \times 5$	1
(ii) $252 = 2^2 \times 3^2 \times 7$	1
(iii) $4725 = 3^3 \times 5^2 \times 7$	1
(iv) $6435 = 3^2 \times 5 \times 11 \times 13$	1
(v) $4199 = 13 \times 17 \times 19$	1

Q	Answer	Working / reason
A2	**(c)** 4	$144 = 2^4 \times 3^2$, so the exponent of 2 is 4.

A3. Check whether 8^n can end with the digit 0 for any natural number n. **[2]**

$8^n = (2^3)^n = 2^{3n}$	½
A number ending in 0 is divisible by $10 = 2 \times 5$, so 5 must be a prime factor	½
By the Fundamental Theorem of Arithmetic the only prime factor of 8^n is 2; 5 is not a factor	½
So 8^n can never end with 0	½

A4. Explain why $5 \times 11 \times 17 + 17$ and $6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6$ are composite numbers. **[2]**

$5 \times 11 \times 17 + 17 = 17\,(5 \times 11 + 1) = 17 \times 56$: it has factors other than 1 and itself, so it is composite	1
$6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6 = 6\,(120 + 1) = 6 \times 121$: composite for the same reason	1

HCF and LCM by prime factorisation

A5. Find the HCF and LCM of 12 and 42 by the prime factorisation method. [2]

$12 = 2^2 \times 3$, $42 = 2 \times 3 \times 7$ — 1

HCF $= 2 \times 3 = 6$ — ½

LCM $= 2^2 \times 3 \times 7 = 84$ — ½

A6. Find the LCM and HCF of each pair and verify that LCM × HCF = product of the two numbers: **(i)** 38 and 57 **(ii)** 390 and 76 **(iii)** 288 and 120 [3]

(i) $38 = 2 \times 19$, $57 = 3 \times 19$: HCF $= 19$, LCM $= 114$; $19 \times 114 = 2166 = 38 \times 57$ ✓ — 1

(ii) $390 = 2 \times 3 \times 5 \times 13$, $76 = 2^2 \times 19$: HCF $= 2$, LCM $= 14820$; $2 \times 14820 = 29640 = 390 \times 76$ ✓ — 1

(iii) $288 = 2^5 \times 3^2$, $120 = 2^3 \times 3 \times 5$: HCF $= 24$, LCM $= 1440$; $24 \times 1440 = 34560 = 288 \times 120$ ✓ — 1

A7. Find the LCM and HCF of the following integers by the prime factorisation method: **(i)** 18, 24 and 30 **(ii)** 13, 19 and 31 **(iii)** 4, 27 and 35 [3]

(i) $18 = 2 \times 3^2$, $24 = 2^3 \times 3$, $30 = 2 \times 3 \times 5$: HCF $= 6$, LCM $= 2^3 \times 3^2 \times 5 = 360$ — 1

(ii) 13, 19, 31 are primes: HCF $= 1$, LCM $= 13 \times 19 \times 31 = 7657$ — 1

(iii) $4 = 2^2$, $27 = 3^3$, $35 = 5 \times 7$: HCF $= 1$, LCM $= 4 \times 27 \times 35 = 3780$ — 1

A8. Given that HCF(252, 594) = 18, find LCM(252, 594). [2]

$$\text{LCM} = \frac{252 \times 594}{\text{HCF}} = \frac{252 \times 594}{18}$$ — 1

$= 14 \times 594 = 8316$ — 1

A9. Two cyclists start together from the same point of a circular track and ride in the same direction. One completes a round in 16 minutes and the other in 20 minutes. After how many minutes will they meet again at the starting point? [2]

They meet when both have done whole rounds: the LCM of 16 and 20 — ½

$16 = 2^4$, $20 = 2^2 \times 5$ — ½

LCM $= 2^4 \times 5 = 80$: they meet after 80 minutes — 1

Irrational numbers

A10. Prove that $\sqrt{5}$ is irrational. [3]

Assume, to the contrary, that $\sqrt{5}$ is rational: $\sqrt{5} = \frac{a}{b}$, where a, b are coprime integers and $b \neq 0$ — ½

Squaring, $5b^2 = a^2$. So 5 divides a^2, and hence 5 divides a — 1

Let $a = 5c$. Then $5b^2 = 25c^2$, i.e. $b^2 = 5c^2$. So 5 divides b^2, and hence 5 divides b — 1

5 is a common factor of a and b, which contradicts that they are coprime. So $\sqrt{5}$ is irrational — ½

A11. Given that $\sqrt{3}$ is irrational, prove that $2 + 5\sqrt{3}$ is irrational. **[3]**

Assume $2 + 5\sqrt{3}$ is rational: $2 + 5\sqrt{3} = \frac{a}{b}$, where a, b are coprime integers, $b \neq 0$ ½

$5\sqrt{3} = \frac{a}{b} - 2 = \frac{a - 2b}{b}$, so $\sqrt{3} = \frac{a - 2b}{5b}$ 1

a, b are integers, so $\frac{a - 2b}{5b}$ is rational, and then $\sqrt{3}$ would be rational 1

This contradicts the fact that $\sqrt{3}$ is irrational. Hence $2 + 5\sqrt{3}$ is irrational ½

A12. Prove that the following are irrational: **(i)** $\frac{1}{\sqrt{3}}$ **(ii)** $4\sqrt{2}$ **(iii)** $7 - \sqrt{5}$ **[3]**

(i) If $\frac{1}{\sqrt{3}} = \frac{a}{b}$ (a, b integers, $a, b \neq 0$), then $\sqrt{3} = \frac{b}{a}$ is rational: contradiction. So $\frac{1}{\sqrt{3}}$ is irrational 1

(ii) If $4\sqrt{2} = \frac{a}{b}$, then $\sqrt{2} = \frac{a}{4b}$ is rational: contradiction. So $4\sqrt{2}$ is irrational 1

(iii) If $7 - \sqrt{5} = \frac{a}{b}$, then $\sqrt{5} = \frac{7b - a}{b}$ is rational: contradiction. So $7 - \sqrt{5}$ is irrational 1

Q	Answer	Working / reason
A13	**(b)** 3 divides a	If a prime p divides a^2, then p divides a. Here $p = 3$. ($a = 3$ and $a = 6$ show that the other options need not be true.)

PART B · SOLUTIONS

B Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(b)** x^2y	Smallest power of each common prime: x^2 and $y^1 \rightarrow x^2y$.
B2	**(b)** an even number	odd + odd = even. ($3 + 5 = 8$, $3 + 7 = 10$; 10 is not a multiple of 4.)
B3	**(c)** 99	Smallest odd composite = 9, smallest two-digit prime = 11; they are coprime, so LCM = 99.
B4	**(a)** 315	$\text{LCM} = \frac{45 \times 105}{15} = 3 \times 105 = 315$
B5	**(d)** 6	$360 = 2^3 \times 3^2 \times 5$: $3 + 2 + 1 = 6$
B6	**(b)** 840	$\text{LCM}(1, \ldots, 8) = 2^3 \times 3 \times 5 \times 7 = 840$
B7	**(a)** 1	$84 = 2^2 \cdot 3 \cdot 7$, $126 = 2 \cdot 3^2 \cdot 7 \rightarrow \text{HCF} = 42$. $126 - 84m = 42 \Rightarrow m = 1$
B8	**(d)** 4	The greatest power of 2 among 2^2, 2^1, 2^n must be 2^4, so $n = 4$.
B9	**(c)** $\sqrt{2} \times \sqrt{8}$	$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$. The others are irrational ($\sqrt{2} + \sqrt{8} = 3\sqrt{2}$).

Short answer with reasons

B10. Can two numbers have 15 as their HCF and 175 as their LCM? Give a reason. [2]

The HCF of two numbers always divides their LCM — 1

$175 \div 15 = 11\frac{2}{3}$ is not a whole number, so no two numbers can have this HCF and LCM — 1

B11. Show that 14^n cannot end with the digit 0 or 5 for any natural number n. [2]

$14^n = 2^n \times 7^n$ — ½

A number ending in 0 or 5 is divisible by 5, so 5 would have to be a prime factor — 1

By uniqueness of prime factorisation, 14^n has only the primes 2 and 7. So it cannot end with 0 or 5 — ½

B12. The HCF of two numbers is 9 and their LCM is 360. If one of the numbers is 45, find the other. [2]

$$\text{Other number} = \frac{\text{HCF} \times \text{LCM}}{45} = \frac{9 \times 360}{45}$$ — 1

$= 72$ — 1

B13. Is the product of two irrational numbers always irrational? Justify your answer with an example. [2]

No. $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$, which is rational — 1

But $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ is irrational. So the product may be rational or irrational — 1

B14. Find the HCF and LCM of $2^4 \times 3^2 \times 5$ and $2^2 \times 3^5 \times 7$. Leave your answers in index form. [2]

$\text{HCF} = 2^2 \times 3^2$ (smallest powers of the common primes 2 and 3) — 1

$\text{LCM} = 2^4 \times 3^5 \times 5 \times 7$ (greatest powers of all the primes) — 1

B15. Find the smallest number which, when divided by 12 and by 18, leaves a remainder 5 in each case. [2]

$12 = 2^2 \times 3$, $18 = 2 \times 3^2$: $\text{LCM} = 2^2 \times 3^2 = 36$ — 1

Required number $= 36 + 5 = 41$ — 1

Short answer

B16. Prove that $\sqrt{2} + \sqrt{5}$ is irrational. [3]

Assume $\sqrt{2} + \sqrt{5} = r$, a rational number ($r > 0$) — ½

$\sqrt{5} = r - \sqrt{2}$. Squaring: $5 = r^2 - 2\sqrt{2}\,r + 2$ — ½

$$\sqrt{2} = \frac{r^2 - 3}{2r}$$ — 1

r is rational, so $\dfrac{r^2 - 3}{2r}$ is rational, so $\sqrt{2}$ would be rational: a contradiction — ½

Hence $\sqrt{2} + \sqrt{5}$ is irrational — ½

B17. Given that $\sqrt{6}$ is irrational, show that $\left(\sqrt{2}+\sqrt{3}\right)^2$ is irrational. [3]

$\left(\sqrt{2}+\sqrt{3}\right)^2 = 2+3+2\sqrt{6} = 5+2\sqrt{6}$ — 1

Assume $5+2\sqrt{6} = \frac{a}{b}$ (a, b coprime integers, $b \neq 0$). Then $\sqrt{6} = \frac{a-5b}{2b}$ — 1

The right side is rational but $\sqrt{6}$ is irrational: contradiction. So $(\sqrt{2}+\sqrt{3})^2$ is irrational — 1

B18. Three friends start walking together from the same point. Their steps measure 63 cm, 70 cm and 77 cm. What is the minimum distance each should walk so that all of them cover it in a whole number of steps? [3]

The distance must be a multiple of 63, 70 and 77: we need their LCM — ½

$63 = 3^2 \times 7$, $70 = 2 \times 5 \times 7$, $77 = 7 \times 11$ — 1

$\text{LCM} = 2 \times 3^2 \times 5 \times 7 \times 11 = 6930$ — 1

Minimum distance $= 6930$ cm $= 69.3$ m — ½

C · Board Pattern Practice – Solutions

PART C · SOLUTIONS

Worked solutions with step marks for every question in Part C.

Irrationality proofs

C1. Prove that $\sqrt{3}$ is irrational. [3]

Assume, to the contrary, that $\sqrt{3} = \frac{a}{b}$, where a, b are coprime integers and $b \neq 0$ — ½

Squaring, $3b^2 = a^2$. So 3 divides a^2, and hence 3 divides a — 1

Let $a = 3c$. Then $3b^2 = 9c^2$, i.e. $b^2 = 3c^2$. So 3 divides b^2, and hence 3 divides b — 1

3 is a common factor of a and b: this contradicts that they are coprime. So $\sqrt{3}$ is irrational — ½

C2. Given that $\sqrt{5}$ is irrational, prove that $4-3\sqrt{5}$ is irrational. [2]

Assume $4-3\sqrt{5} = \frac{a}{b}$, where a, b are coprime integers, $b \neq 0$ — ½

$3\sqrt{5} = 4 - \frac{a}{b} = \frac{4b-a}{b}$, so $\sqrt{5} = \frac{4b-a}{3b}$ — 1

The right side is rational, so $\sqrt{5}$ would be rational: contradiction. Hence $4-3\sqrt{5}$ is irrational — ½

C3. Given that $\sqrt{5}$ is irrational, prove that $\frac{3}{\sqrt{5}}$ is irrational. [2]

Assume $\frac{3}{\sqrt{5}} = \frac{a}{b}$, where a, b are coprime integers, $a \neq 0, b \neq 0$ — ½

Then $\sqrt{5} = \frac{3b}{a}$ — 1

$\frac{3b}{a}$ is rational, so $\sqrt{5}$ would be rational: contradiction. Hence $\frac{3}{\sqrt{5}}$ is irrational — ½

C4. Given that $\sqrt{3}$ is irrational, prove that $\frac{5+2\sqrt{3}}{7}$ is irrational. [3]

Assume $\frac{5+2\sqrt{3}}{7} = \frac{a}{b}$, where a, b are coprime integers, $b \neq 0$ ½

$5+2\sqrt{3} = \frac{7a}{b}$, so $2\sqrt{3} = \frac{7a-5b}{b}$ 1

$\sqrt{3} = \frac{7a-5b}{2b}$, which is rational as a, b are integers 1

This contradicts that $\sqrt{3}$ is irrational. Hence $\frac{5+2\sqrt{3}}{7}$ is irrational ½

C5. Given that $\sqrt{2}$ is irrational, prove that $3\sqrt{2}-1$ is irrational. [2]

Assume $3\sqrt{2}-1 = \frac{a}{b}$, where a, b are coprime integers, $b \neq 0$ ½

$3\sqrt{2} = \frac{a+b}{b}$, so $\sqrt{2} = \frac{a+b}{3b}$ 1

The right side is rational, so $\sqrt{2}$ would be rational: contradiction. Hence $3\sqrt{2}-1$ is irrational ½

Q	Answer	Working / reason
C6	**(c)**	A is true: $\sqrt{3}+\sqrt{12} = \sqrt{3}+2\sqrt{3} = 3\sqrt{3}$, irrational. R is false: $\sqrt{2}+(-\sqrt{2}) = 0$.

Finding HCF and LCM

Q	Answer	Working / reason
C7	**(b)** 252	$756 = 2^2 \times 3^3 \times 7$, $504 = 2^3 \times 3^2 \times 7 \rightarrow$ HCF $= 2^2 \times 3^2 \times 7 = 252$
C8	**(d)** 216	Greatest powers: $2^3 \times 3^3 = 216$
C9	**(b)** $30a^4b^3$	$15 = 3 \cdot 5$, $10 = 2 \cdot 5 \rightarrow 2 \cdot 3 \cdot 5 = 30$; greatest powers a^4, $b^3 \rightarrow 30a^4b^3$
C10	**(c)** 150	LCM $= \frac{1800}{12} = 150$
C11	**(a)** 0	$56 = 2^3 \cdot 7$, $84 = 2^2 \cdot 3 \cdot 7$: $m = 28$, $n = 2^3 \cdot 3 \cdot 7 = 168$. $168 - 168 = 0$
C12	**(c)** pq	Coprime $\rightarrow$ HCF $= 1$, so LCM $= \frac{pq}{1} = pq$.
C13	**(c)** 100	LCM $= \frac{1080 \times 1200}{120} = 10800 = 108 \times 100$
C14	**(d)**	$380 \div 16 = 23.75$ is not whole, so A is false. R is true.

C15. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other. [2]

Other number $= \frac{23 \times 1449}{161}$ 1

$= \frac{33327}{161} = 207$ 1

C16. Find the HCF of 120 and 144 by prime factorisation. Hence find their LCM. [2]

$120 = 2^3 \times 3 \times 5$, $144 = 2^4 \times 3^2$ ½

HCF $= 2^3 \times 3 = 24$ ½

LCM $= \frac{120 \times 144}{24} = 720$ 1

C17. Two numbers are in the ratio 3 : 5 and their LCM is 240. Find their HCF. [2]

Let the numbers be $3k$ and $5k$. As 3 and 5 are coprime, HCF $= k$ and LCM $= 15k$ — 1

$15k = 240 \Rightarrow k = 16$: HCF $= 16$ (numbers 48 and 80) — 1

C18. Find the HCF and LCM of 72, 108 and 180 by prime factorisation. Is HCF $\times$ LCM equal to the product of the three numbers? Justify. [3]

$72 = 2^3 \times 3^2$, $108 = 2^2 \times 3^3$, $180 = 2^2 \times 3^2 \times 5$ — 1

HCF $= 2^2 \times 3^2 = 36$ — ½

LCM $= 2^3 \times 3^3 \times 5 = 1080$ — ½

HCF × LCM $= 38880$ but $72 \times 108 \times 180 = 1399680$. Not equal: the rule holds for two numbers only — 1

HCF and LCM word problems

C19. Three school bells ring at intervals of 12, 18 and 30 minutes. If they ring together at 8:00 a.m., at what time will they next ring together? [2]

$12 = 2^2 \times 3$, $18 = 2 \times 3^2$, $30 = 2 \times 3 \times 5$: LCM $= 2^2 \times 3^2 \times 5 = 180$ min — 1

180 min = 3 hours, so they ring together again at 11:00 a.m. — 1

C20. The traffic lights at three crossings change every 48 seconds, 72 seconds and 108 seconds. If they change together at 7:00:00 a.m., at what time will they next change together? [2]

$48 = 2^4 \times 3$, $72 = 2^3 \times 3^2$, $108 = 2^2 \times 3^3$: LCM $= 2^4 \times 3^3 = 432$ s — 1

432 s = 7 min 12 s, so at 7:07:12 a.m. — 1

C21. A school workshop has 45 Science, 75 Maths and 105 English teachers. Each room must seat the same number of teachers, all of one subject. Find the minimum number of rooms needed. [3]

Fewest rooms → most teachers per room, a number dividing 45, 75 and 105: the HCF — ½

$45 = 3^2 \times 5$, $75 = 3 \times 5^2$, $105 = 3 \times 5 \times 7$ — 1

HCF $= 3 \times 5 = 15$ teachers per room — ½

Rooms $= \dfrac{45 + 75 + 105}{15} = \dfrac{225}{15} = 15$ — 1

C22. A sweet shop has 576 laddoos and 432 barfis. They are to be stacked so that each stack has the same number of sweets, all of one kind, and the stacks cover the least area of the tray. How many sweets go in each stack? How many stacks are there? [3]

Least area → fewest stacks → most sweets per stack: HCF(576, 432) — ½

$576 = 2^6 \times 3^2$, $432 = 2^4 \times 3^3$ — 1

HCF $= 2^4 \times 3^2 = 144$ sweets per stack — ½

Stacks $= \dfrac{576}{144} + \dfrac{432}{144} = 4 + 3 = 7$ — 1

C23. The floor of a hall is 15 m 17 cm long and 9 m 2 cm broad. It is to be paved with equal square tiles. Find the side of the largest square tile that can be used and the number of tiles needed. [3]

In cm: 1517 cm and 902 cm. The side of the tile must divide both: HCF ½

$1517 = 37 \times 41$, $902 = 2 \times 11 \times 41 \rightarrow$ HCF $= 41$ cm 1

$\text{Tiles} = \dfrac{1517 \times 902}{41 \times 41} = 37 \times 22$ 1

$= 814$ tiles ½

C24. Find the greatest length of a measuring rod that can measure 3 m 85 cm and 5 m 95 cm an exact number of times. [2]

385 cm and 595 cm: $385 = 5 \times 7 \times 11$, $595 = 5 \times 7 \times 17$ 1

HCF $= 5 \times 7 = 35$: the rod is 35 cm long 1

C25. Find the greatest number which divides 245 and 1029, leaving a remainder 5 in each case. [2]

The number divides $245 - 5 = 240$ and $1029 - 5 = 1024$ exactly ½

$240 = 2^4 \times 3 \times 5$, $1024 = 2^{10}$ ½

HCF $= 2^4 = 16$ 1

Q	Answer	Working / reason
C26	**(b)** 105	HCF$(315, 735)$: $315 = 3^2 \cdot 5 \cdot 7$, $735 = 3 \cdot 5 \cdot 7^2 \rightarrow 3 \cdot 5 \cdot 7 = 105$

C27. Find the smallest number which, when increased by 13, is exactly divisible by both 280 and 504. [2]

$280 = 2^3 \times 5 \times 7$, $504 = 2^3 \times 3^2 \times 7$: LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$ 1

Required number $= 2520 - 13 = 2507$ 1

Prime and composite numbers · factor trees

Q	Answer	Working / reason
C28	**(c)** 2 is the only even prime number	1 is neither prime nor composite; 4 is the smallest composite; 9 is odd but not prime.
C29	**(b)** a composite number	$7(5 \times 11 + 1) = 7 \times 56 = 392$: it has factors other than 1 and itself.

C30. Show that $2 \times 3 \times 5 \times 7 \times 11 + 7 \times 11$ is a composite number. [2]

$= 7 \times 11\,(2 \times 3 \times 5 + 1) = 7 \times 11 \times 31$ 1

It has factors 7, 11, 31 other than 1 and itself, so it is composite 1

Q	Answer	Working / reason
C31	**(a)**	$13(77 + 1) = 13 \times 78$, so A is true, and R is exactly the reason.
C32	**(a)** coprime	a^2 and b^2 have the same primes as a and b, so they share no prime factor.

C33. In a factor tree, N splits as $2 \times a$, a splits as $3 \times b$, and b splits as 7×13. Find b, a and N, and write N as a product of primes. [2]

$b = 7 \times 13 = 91$; $a = 3 \times 91 = 273$ — 1

$N = 2 \times 273 = 546 = 2 \times 3 \times 7 \times 13$ — 1

Last digit · numbers ending in 0

Q	Answer	Working / reason
C34	**(a)** 1	5^n always ends in 5 and 6^n always ends in 6; $5 + 6 = 11 \rightarrow$ last digit 1.
C35	**(d)** 20^n	Only $20^n = 2^{2n} \times 5^n$ has both 2 and 5 as factors.
C36	**(b)** 1	…6 – …5 → last digit 1.
C37	**(a)** 2	Each zero needs one pair 2×5. There are only two 5s, so 2 zeros.

C38. Check whether 35^n can end with the digit 0 for any natural number n. Justify your answer. [2]

$35^n = 5^n \times 7^n$ — ½

To end with 0 a number must have both 2 and 5 as prime factors — ½

2 is not a prime factor of 35^n (prime factorisation is unique), so 35^n never ends with 0 — 1

Rational or irrational? Number facts

Q	Answer	Working / reason
C39	**(b)** 1.5	$\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$: only 1.5 lies between them and is rational ($\sqrt{2.5}$ is irrational).
C40	**(c)** a rational number	$\frac{45}{80} = \frac{9}{16}$, so $p = \pm\frac{3}{4}$: rational.
C41	**(d)** $(1+\sqrt{3})^2$	$(1+\sqrt{3})^2 = 4 + 2\sqrt{3}$. The others are 1, 5 and 6.
C42	**(a)**	Both are true and R is the reason for A.
C43	**(b)** n is odd	$(-1)^{2n} = 1$, so we need $(-1)^n = -1$: n is odd.

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Sports Meet [4]

(i) After how many minutes will Aarav and Bhavya first meet again at the starting point? **[1]**

$\text{LCM}(6, 9) = 2 \times 3^2 = 18$ minutes — 1

(ii) Find the HCF of 6, 9 and 15. **[1]**

$6 = 2 \times 3$, $9 = 3^2$, $15 = 3 \times 5$: $\text{HCF} = 3$ — 1

(iii) At what time will all three be at the starting point together again? **[2]**

$\text{LCM}(6, 9, 15) = 2 \times 3^2 \times 5 = 90$ minutes — 1

90 min = 1 h 30 min after 7:00 → 8:30 a.m. — 1

OR

(iii) How many rounds will Charu have completed when all three first meet again at the starting point? **[2]**

They meet after LCM = 90 minutes — 1

Charu's rounds $= \dfrac{90}{15} = 6$ — 1

D2. Case Study · Donation Drive [4]

(i) What is the greatest number of identical kits they can make? **[1]**

$336 = 2^4 \cdot 3 \cdot 7$, $240 = 2^4 \cdot 3 \cdot 5$, $96 = 2^5 \cdot 3$: $\text{HCF} = 2^4 \times 3 = 48$ kits — 1

(ii) How many notebooks will there be in each kit? **[1]**

$\dfrac{336}{48} = 7$ notebooks — 1

(iii) Write 336 and 240 as products of primes and find their LCM. **[2]**

$336 = 2^4 \times 3 \times 7$, $240 = 2^4 \times 3 \times 5$ — 1

$\text{LCM} = 2^4 \times 3 \times 5 \times 7 = 1680$ — 1

OR

(iii) If they had collected 252 pencils instead of 240, what would be the greatest number of identical kits? **[2]**

$252 = 2^2 \times 3^2 \times 7$ — 1

$\text{HCF}(336, 252, 96) = 2^2 \times 3 = 12$ kits — 1

D3. Case Study · Number Card Game [4]

(i) Which of the cards A to F show rational numbers? **[1]**

$\text{A} = 6$, $\text{B} = \sqrt{100} = 10$, $\text{E} = \dfrac{3}{4}$: cards A, B and E — 1

(ii) Is the sum of the numbers on cards C and F rational or irrational? Give a reason. **[1]**

$2 + \sqrt{3} + 2\sqrt{3} = 2 + 3\sqrt{3}$: rational + irrational, so irrational — 1

(iii) Given that $\sqrt{3}$ is irrational, prove that the number on card C is irrational. **[2]**

Assume $2+\sqrt{3}=\frac{a}{b}$ (a, b coprime integers, $b\neq 0$). Then $\sqrt{3}=\frac{a-2b}{b}$ — 1

The right side is rational: contradiction, as $\sqrt{3}$ is irrational. So $2+\sqrt{3}$ is irrational — 1

OR

(iii) Given that $\sqrt{7}$ is irrational, prove that the number on card D is irrational. **[2]**

Assume $4\sqrt{7}=\frac{a}{b}$ (a, b coprime integers, $b\neq 0$). Then $\sqrt{7}=\frac{a}{4b}$ — 1

The right side is rational: contradiction, as $\sqrt{7}$ is irrational. So $4\sqrt{7}$ is irrational — 1

D4. Case Study · Factor Tree [4]

(i) Find the values of c and b. **[1]**

$c=7\times 11=77; b=5\times 77=385$ — 1

(ii) Find N and write it as a product of primes. **[1]**

$a=3\times 385=1155; N=2\times 1155=2310=2\times 3\times 5\times 7\times 11$ — 1

(iii) Find the HCF and LCM of N and 840 by prime factorisation. **[2]**

$840=2^3\times 3\times 5\times 7, N=2\times 3\times 5\times 7\times 11$ — ½

$\text{HCF}=2\times 3\times 5\times 7=210$ — ½

$\text{LCM}=2^3\times 3\times 5\times 7\times 11=9240$ — 1

OR

(iii) Find the smallest number divisible by each of 2, 3, 4, 5, 7 and 11. Is it equal to N? Give a reason. **[2]**

$\text{LCM}=2^2\times 3\times 5\times 7\times 11=4620$ — 1

No: $4620=2\times 2310$. N has only one factor 2, so it is not divisible by 4 — 1

E PART E · SOLUTIONS
Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. [3]

If p is a prime number, prove that $\sqrt{p}$ is irrational.

Assume $\sqrt{p}=\frac{a}{b}$, where a, b are coprime integers, $b\neq 0$ — ½

$pb^2=a^2$, so p divides a^2, hence p divides a — 1

Let $a=pc$: $pb^2=p^2c^2\Rightarrow b^2=pc^2$, so p divides b^2, hence p divides b — 1

p is a common factor of a and b: contradiction. So $\sqrt{p}$ is irrational — ½

E2. [3]

If p and q are two different primes, prove that $\sqrt{p}+\sqrt{q}$ is irrational.

Assume $\sqrt{p}+\sqrt{q}=r$, rational and $r>0$ — ½

$\sqrt{q}=r-\sqrt{p}\Rightarrow q=r^2-2r\sqrt{p}+p$ — ½

$\sqrt{p}=\frac{r^2+p-q}{2r}$, which is rational — 1

But $\sqrt{p}$ is irrational for a prime p (previous question): contradiction. So $\sqrt{p}+\sqrt{q}$ is irrational — 1

E3. Find the smallest number which, when divided by 28 and 32, leaves remainders 8 and 12 respectively. [3]

$28 - 8 = 20$ and $32 - 12 = 20$: the number is 20 short of a common multiple of 28 and 32 — 1

$28 = 2^2 \times 7$, $32 = 2^5$: LCM $= 2^5 \times 7 = 224$ — 1

Number $= 224 - 20 = 204$ (check: $204 = 7 \times 28 + 8 = 6 \times 32 + 12$) — 1

E4. Find the least number which leaves a remainder 5 when divided by 6, 15 and 18, and is exactly divisible by 7. [3]

$\text{LCM}(6, 15, 18) = 2 \times 3^2 \times 5 = 90$, so the number is of the form $90k + 5$ — 1

$k = 1, 2, 3, 4$ give 95, 185, 275, 365: none is divisible by 7 — 1

$k = 5$: $455 = 7 \times 65$. The number is 455 — 1

E5. The HCF of two numbers is 6 and their product is 1080. Find their LCM, and all possible pairs of such numbers. [3]

$\text{LCM} = \dfrac{1080}{6} = 180$ — 1

Numbers $6m$, $6n$ with m, n coprime: $36mn = 1080 \Rightarrow mn = 30$ — 1

Coprime pairs $(1, 30), (2, 15), (3, 10), (5, 6) \rightarrow$ (6, 180), (12, 90), (18, 60), (30, 36) — 1

E6. Find the greatest 4-digit number which is exactly divisible by 15, 24 and 36. [2]

$\text{LCM}(15, 24, 36) = 2^3 \times 3^2 \times 5 = 360$ — 1

$9999 = 27 \times 360 + 279$, so the number is $9999 - 279 = 9720$ — 1

E7. Find the smallest 5-digit number which is exactly divisible by 16, 24 and 40. [2]

$\text{LCM}(16, 24, 40) = 2^4 \times 3 \times 5 = 240$ — 1

$10000 = 41 \times 240 + 160$, so the number is $42 \times 240 = 10080$ — 1