Affiliated to CBSE, New Delhi (Affiliation No. 1030642)

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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 9

Some Applications of Trigonometry

30°
45°
60°

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check your answer with the quick facts: at 30° the distance is $\sqrt{3}$ times the height, at 60° it is the height divided by $\sqrt{3}$.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN SOME APPLICATIONS OF TRIGONOMETRY ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Long answer (5 marks):** correct figure 1 · first trigonometric equation 1 · second equation 1 · solving 1 · answer with unit 1.
3. **3-mark question:** figure with the first equation 1 · the second equation or working 1 · answer 1.
4. **Case study 2-mark part:** the equation 1 · the answer 1.
5. An exact answer such as $20\sqrt{3}$ m scores full marks unless a decimal is asked for.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

One right triangle

A1. A rope 16 m long is stretched tight from the top of a vertical pole to a peg on the ground. It makes an angle of 30° with the ground. Find the height of the pole. [2]

$\sin 30^\circ = \dfrac{h}{16}$ 1

$h = 16 \times \dfrac{1}{2} = 8$ m 1

A2. A storm breaks a tree. The top part bends over and its tip touches the ground 9 m from the foot of the tree, making 30° with the ground. Find the height of the tree before it broke. [3]

Standing part $BC = 9\tan 30^\circ = \dfrac{9}{\sqrt{3}} = 3\sqrt{3}$ m 1

Bent part $AC = \dfrac{9}{\cos 30^\circ} = \dfrac{18}{\sqrt{3}} = 6\sqrt{3}$ m 1

Height $= 3\sqrt{3} + 6\sqrt{3} = 9\sqrt{3} \approx 15.59$ m 1

A3. A park has two slides. Find the length of each slide. **(i)** for small children: the top is 1.8 m high and the slide makes 30° with the ground **(ii)** for older children: the top is 3 m high and the slide makes 60° with the ground [2]

(i) $\sin 30^\circ = \dfrac{1.8}{L} \rightarrow L = 3.6$ m 1

(ii) $\sin 60^\circ = \dfrac{3}{L} \rightarrow L = \dfrac{6}{\sqrt{3}} = 2\sqrt{3} \approx 3.46$ m 1

A4. The angle of elevation of the top of a tower from a point on the ground 45 m from its foot is 30°. Find the height of the tower. [2]

$\tan 30^\circ = \dfrac{h}{45}$ 1

$h = \dfrac{45}{\sqrt{3}} = 15\sqrt{3} \approx 25.98$ m 1

A5. A kite flies 75 m above the ground. Its string, tied to a point on the ground, makes 60° with the ground. Find the length of the string, taking it to be straight. [2]

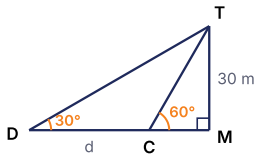
$\sin 60^\circ = \dfrac{75}{L}$ 1

$L = \dfrac{150}{\sqrt{3}} = 50\sqrt{3} \approx 86.6$ m 1

Q	Answer	Working / reason
A6	**(c)** 60°	$\tan\theta = \dfrac{12}{4\sqrt{3}} = \sqrt{3}$, so $\theta = 60^\circ$.
A7	**(a)** 5 m	$10\cos 60^\circ = 5$ m.

Two right triangles

A8. A boy whose eyes are 1.6 m above the ground stands some distance from a building 31.6 m high. As he walks towards the building, the angle of elevation of its top from his eyes changes from 30° to 60°. How far did he walk? [3]

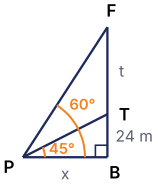


Height above eye level $TM = 31.6 - 1.6 = 30$ m 1

$DM = \dfrac{30}{\tan 30^\circ} = 30\sqrt{3}$ m, $CM = \dfrac{30}{\tan 60^\circ} = 10\sqrt{3}$ m 1

Distance walked $DC = 20\sqrt{3} \approx 34.64$ m 1

A9. From a point on the ground, the angles of elevation of the bottom and the top of a mobile tower fixed on a 24 m high building are 45° and 60°. Find the height of the tower. [3]

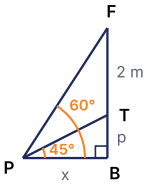


$\tan 45^\circ = \dfrac{24}{x} \rightarrow x = 24$ m 1

$\tan 60^\circ = \dfrac{24 + t}{24} \rightarrow 24 + t = 24\sqrt{3}$ 1

$t = 24(\sqrt{3} - 1) \approx 17.57$ m 1

A10. A statue 2 m tall stands on a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and that of the top of the pedestal is 45°. Find the height of the pedestal. **[3]**

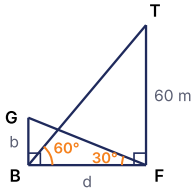


$\tan 45^\circ = \dfrac{p}{x} \to x = p$ — 1

$\tan 60^\circ = \dfrac{p+2}{p} = \sqrt{3} \to p(\sqrt{3}-1) = 2$ — 1

$p = \dfrac{2}{\sqrt{3}-1} = \sqrt{3}+1 \approx 2.73$ m — 1

A11. The angle of elevation of the top of a building from the foot of a tower is 30°, and the angle of elevation of the top of the tower from the foot of the building is 60°. The tower is 60 m high. Find the height of the building. **[3]**

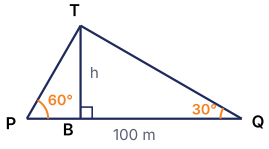


$\tan 60^\circ = \dfrac{60}{d} \to d = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ m — 1

$\tan 30^\circ = \dfrac{b}{d} \to b = 20\sqrt{3} \times \dfrac{1}{\sqrt{3}}$ — 1

$b = 20$ m — 1

A12. Two poles of equal height stand exactly opposite each other on the two sides of a road 100 m wide. From a point on the road between them, the angles of elevation of their tops are 60° and 30°. Find the height of the poles and the distances of the point from them. **[4]**



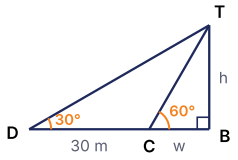
Let the point be x m from the pole seen at 60°: $h = x\tan 60^\circ = \sqrt{3}x$ — 1

$h = (100-x)\tan 30^\circ = \dfrac{100-x}{\sqrt{3}}$ — 1

$3x = 100 - x \to x = 25$ — 1

$h = 25\sqrt{3} \approx 43.3$ m; the point is 25 m and 75 m from the poles — 1

A13. A TV tower stands on one bank of a canal. From a point on the other bank, directly opposite the tower, the angle of elevation of its top is 60°. From a point 30 m further back on the same line, the angle is 30°. Find the height of the tower and the width of the canal. **[4]**



Width w: $h = w\tan 60^\circ = \sqrt{3}w$ — 1

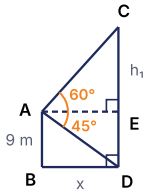
$h = (w+30)\tan 30^\circ = \dfrac{w+30}{\sqrt{3}}$ — 1

$3w = w + 30 \to w = 15$ m — 1

$h = 15\sqrt{3} \approx 25.98$ m — 1

Angles of depression

A14. From the top of a 9 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. [3]

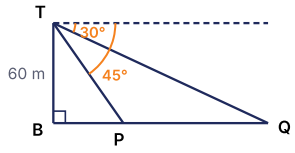


$\tan 45^\circ = \dfrac{9}{x} \to x = 9$ m — 1

$\tan 60^\circ = \dfrac{h_1}{9} \to h_1 = 9\sqrt{3}$ m — 1

Tower $= 9 + 9\sqrt{3} = 9(1+\sqrt{3}) \approx 24.59$ m — 1

A15. From the top of a 60 m high lighthouse, the angles of depression of two ships on the same side of it, in line with its foot, are 30° and 45°. Find the distance between the ships. [3]

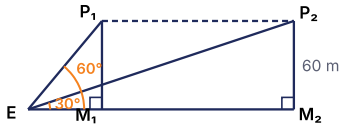


Nearer ship: $\tan 45^\circ = \dfrac{60}{BP} \to BP = 60$ m — 1

Farther ship: $\tan 30^\circ = \dfrac{60}{BQ} \to BQ = 60\sqrt{3}$ m — 1

$PQ = 60(\sqrt{3}-1) \approx 43.92$ m — 1

A16. A girl whose eyes are 1.5 m above the ground sees a balloon drifting horizontally at a height of 61.5 m above the ground. The angle of elevation of the balloon from her eyes is 60°. Some time later it is 30°. How far did the balloon move? [3]

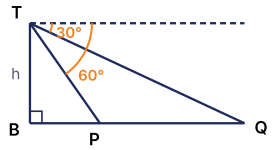


Height above eye level $= 61.5 - 1.5 = 60$ m — 1

$EM_1 = \dfrac{60}{\tan 60^\circ} = 20\sqrt{3}$ m, $EM_2 = \dfrac{60}{\tan 30^\circ} = 60\sqrt{3}$ m — 1

Distance moved $= 40\sqrt{3} \approx 69.28$ m — 1

A17. A straight road leads to the foot of a tower. A man at the top sees a car at an angle of depression of 30°, coming towards the tower at a steady speed. Eight seconds later the angle of depression is 60°. How much longer will the car take to reach the foot of the tower? [3]



Tower h: $BQ = \dfrac{h}{\tan 30^\circ} = \sqrt{3}h$, $BP = \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$ — 1

$QP = \sqrt{3}h - \dfrac{h}{\sqrt{3}} = \dfrac{2h}{\sqrt{3}}$ takes 8 s — 1

$BP = \dfrac{h}{\sqrt{3}}$ is half of QP, so it takes 4 s more — 1

Q	Answer	Working / reason
A18	**(c)** 20 m	The angle at the car is 45°, so the distance $= \dfrac{20}{\tan 45^\circ} = 20$ m.

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(b)** $45°$	$\tan\theta = 1$, so $\theta = 45^\circ$.
B2	**(b)** $10\sqrt{3}$ m	The distances $h\sqrt{3}$ and $\frac{h}{\sqrt{3}}$ differ by $\frac{2h}{\sqrt{3}} = 20$, so $h = 10\sqrt{3}$ m.
B3	**(d)** $24\sqrt{3}$ m	$\frac{24}{\tan 30^\circ} = 24\sqrt{3}$ m.
B4	**(c)** $10\sqrt{3}$ m	$20\sin 60^\circ = 10\sqrt{3}$ m.
B5	**(c)** $\sqrt{ab}$	$h = \frac{a}{\sqrt{3}}$ and $h = \sqrt{3}b$. Multiply: $h^2 = ab$.
B6	**(a)**	Both true, and R explains A.
B7	**(b)**	A is true: $\tan\theta = \frac{1}{\sqrt{3}}$. R is true, but A uses tan, not sin, so R does not explain A.
B8	**(d)** $30°$	$\tan\theta = \frac{1}{\sqrt{3}}$, so $\theta = 30^\circ$.

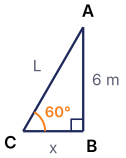
Short answer with reasons

B9. The angle of elevation of the top of a 15 m pole from a point P on the ground is 45°. How far must P move towards the pole for the angle to become 60°? **[2]**

At 45° the distance is 15 m; at 60° it is $\frac{15}{\sqrt{3}} = 5\sqrt{3}$ m — 1

Move $15 - 5\sqrt{3} \approx 15 - 8.66 = 6.34$ m — 1

B10. A painter must reach a point 1.5 m below the top of a 7.5 m wall. The ladder must make 60° with the ground. Find the length of the ladder and the distance of its foot from the wall. **[3]**



Height to reach $= 7.5 - 1.5 = 6$ m — 1

$\sin 60^\circ = \frac{6}{L} \rightarrow L = \frac{12}{\sqrt{3}} = 4\sqrt{3} \approx 6.93$ m — 1

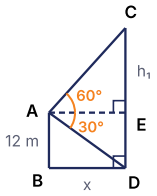
$\tan 60^\circ = \frac{6}{x} \rightarrow x = 2\sqrt{3} \approx 3.46$ m — 1

B11. The shadow of a tower is 40 m longer when the Sun's elevation is 30° than when it is 60°. Find the height of the tower. **[2]**

Shadows: $\sqrt{3}h$ at 30° and $\frac{h}{\sqrt{3}}$ at 60°; difference $\frac{2h}{\sqrt{3}} = 40$ — 1

$h = 20\sqrt{3} \approx 34.64$ m — 1

B12. From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Find the height of the tower. **[3]**



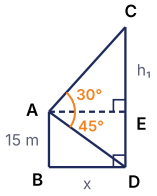
$\tan 30^\circ = \dfrac{12}{x} \rightarrow x = 12\sqrt{3}$ m — 1

$\tan 60^\circ = \dfrac{h_1}{12\sqrt{3}} \rightarrow h_1 = 36$ m — 1

Tower $= 12 + 36 = 48$ m — 1

Long answer

B13. From a window 15 m above the ground, the angle of elevation of the top of a house across the street is 30° and the angle of depression of its foot is 45°. Show that the height of the house is $5(3+\sqrt{3})$ m. **[4]**



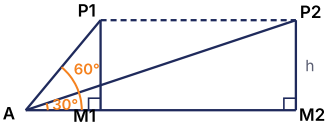
Correct figure — 1

$\tan 45^\circ = \dfrac{15}{x} \rightarrow x = 15$ m — 1

$\tan 30^\circ = \dfrac{h_1}{15} \rightarrow h_1 = 5\sqrt{3}$ m — 1

Height $= 15 + 5\sqrt{3} = 5(3+\sqrt{3}) \approx 23.66$ m — 1

B14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 24 seconds at a constant height of $1200\sqrt{3}$ m, the angle of elevation from A is 30°. Find the speed of the plane in km/h. **[5]**



Correct figure — 1

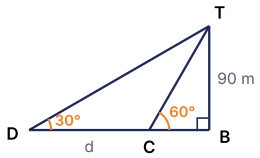
$AM_1 = \dfrac{1200\sqrt{3}}{\tan 60^\circ} = 1200$ m — 1

$AM_2 = \dfrac{1200\sqrt{3}}{\tan 30^\circ} = 3600$ m — 1

Distance flown $= 3600 - 1200 = 2400$ m in 24 s — 1

Speed $= 100$ m/s $= 360$ km/h — 1

B15. A boat is rowed straight away from a cliff 90 m high. In 3 minutes the angle of elevation of the top of the cliff from the boat changes from 60° to 30°. Find the speed of the boat in m/min. **[5]**



Correct figure — 1

$CB = \dfrac{90}{\tan 60^\circ} = 30\sqrt{3}$ m — 1

$DB = \dfrac{90}{\tan 30^\circ} = 90\sqrt{3}$ m — 1

Distance rowed $d = 60\sqrt{3}$ m in 3 minutes — 1

Speed $= 20\sqrt{3} \approx 34.64$ m/min — 1

C Board Pattern Practice – Solutions

PART C · SOLUTIONS

Worked solutions with step marks for every question in Part C.

One right triangle: a height, a length or an angle

Q	Answer	Working / reason
C1	**(c)** 60°	$\tan\theta = \dfrac{24}{8\sqrt{3}} = \sqrt{3}$, so $\theta = 60^\circ$.
C2	**(a)** 40 m	$80\sin 30^\circ = 40$ m.
C3	**(b)** 7 m	$\cos 60^\circ = \dfrac{3.5}{L} \to L = 7$ m.
C4	**(d)** 36 m	$\tan\theta = \dfrac{12}{5}$ gives $\sin\theta = \dfrac{12}{13}$, so the height $= 39 \times \dfrac{12}{13} = 36$ m.
C5	**(d)**	$\tan\theta = \dfrac{30}{30} = 1$, so the angle is 45°: A is false. R is true.

C6. A tree 12 m high casts a shadow. Find the length of the shadow when the Sun's elevation is (i) 30° (ii) 60°. **[2]**

(i) $\dfrac{12}{\tan 30^\circ} = 12\sqrt{3} \approx 20.78$ m — 1

(ii) $\dfrac{12}{\tan 60^\circ} = 4\sqrt{3} \approx 6.93$ m — 1

C7. A vertical pole 10 m high is held by a wire from its top to a point on the ground. The wire makes 45° with the ground. Find the length of the wire and the distance of the point from the foot of the pole. **[2]**

$\sin 45^\circ = \dfrac{10}{L} \to L = 10\sqrt{2} \approx 14.14$ m — 1

$\tan 45^\circ = \dfrac{10}{x} \to x = 10$ m — 1

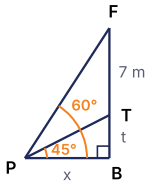
Two right triangles: two angles, or a pole on a tower

C8. The angle of elevation of the top of a tower from a point 4 m from its foot is 60°. Find the angle of elevation of its top from a point 12 m from its foot, on the same line. **[2]**

$h = 4\tan 60^\circ = 4\sqrt{3}$ m — 1

$\tan\phi = \dfrac{4\sqrt{3}}{12} = \dfrac{1}{\sqrt{3}} \to \phi = 30^\circ$ — 1

C9. A flagstaff 7 m long is fixed on top of a tower. From a point on the ground, the angles of elevation of the top of the flagstaff and the top of the tower are 60° and 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$.) **[5]**



Correct figure — 1

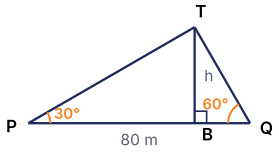
$\tan 45^\circ = \dfrac{t}{x} \to x = t$ — 1

$\tan 60^\circ = \dfrac{t+7}{x} \to t + 7 = \sqrt{3}t$ — 1

$t = \dfrac{7}{\sqrt{3}-1} = \dfrac{7(\sqrt{3}+1)}{2}$ — 1

$t \approx \dfrac{7 \times 2.732}{2} \approx 9.56$ m — 1

C10. Rhea and Kabir stand on level ground on opposite sides of a lamp post, in line with its foot and 80 m apart. The angles of elevation of the top of the post from Rhea and Kabir are 30° and 60°. Find the height of the post and the distance of each of them from it. **[5]**



Correct figure — 1

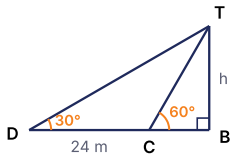
$\tan 30^\circ = \dfrac{h}{PB} \to PB = \sqrt{3}h$ — 1

$\tan 60^\circ = \dfrac{h}{BQ} \to BQ = \dfrac{h}{\sqrt{3}}$ — 1

$\sqrt{3}h + \dfrac{h}{\sqrt{3}} = 80 \to \dfrac{4h}{\sqrt{3}} = 80 \to h = 20\sqrt{3} \approx 34.64$ m — 1

Rhea is $PB = 60$ m and Kabir is $BQ = 20$ m from the post — 1

C11. A vertical mast is held by two straight wires from its top to two points on the ground, on the same side of the mast and in line with its foot. The wires make 30° and 60° with the ground, and the two ground points are 24 m apart. Find the height of the mast and the length of each wire. **[5]**



Correct figure — 1

$DB = \dfrac{h}{\tan 30^\circ} = \sqrt{3}h$, $CB = \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$ — 1

$\sqrt{3}h - \dfrac{h}{\sqrt{3}} = 24 \rightarrow \dfrac{2h}{\sqrt{3}} = 24 \rightarrow h = 12\sqrt{3} \approx 20.78$ m — 1

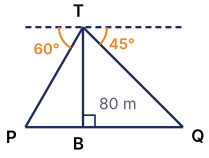
Wire at 30°: $TD = \dfrac{h}{\sin 30^\circ} = 24\sqrt{3} \approx 41.57$ m — 1

Wire at 60°: $TC = \dfrac{h}{\sin 60^\circ} = 24$ m — 1

Angles of depression

Q	Answer	Working / reason
C12	**(b)** ∠XAC	The angle of depression lies between the horizontal AX and the line of sight AC.
C13	**(d)** $\dfrac{50\sqrt{3}}{3}$ m	$\dfrac{50}{\tan 60^\circ} = \dfrac{50}{\sqrt{3}} = \dfrac{50\sqrt{3}}{3}$ m.

C14. From the top of an 80 m high lighthouse, the angles of depression of two boats on opposite sides of it, in line with its foot, are 60° and 45°. (Use $\sqrt{3} = 1.732$.) **(i)** Find the distance between the boats. **(ii)** The boat seen at 60° sails straight away from the lighthouse until its angle of depression is 30°. How far does it sail? **[5]**



Correct figure — 1

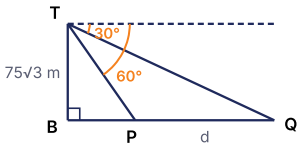
Boat P (60°): $BP = \dfrac{80}{\tan 60^\circ} = \dfrac{80}{\sqrt{3}} \approx 46.19$ m — 1

Boat Q (45°): $BQ = \dfrac{80}{\tan 45^\circ} = 80$ m — 1

(i) Distance between the boats $= 80 + 46.19 \approx 126.19$ m — 1

(ii) Its new distance is $80\sqrt{3}$ m, so it sails $80\sqrt{3} - \dfrac{80}{\sqrt{3}} = \dfrac{160}{\sqrt{3}} \approx 92.38$ m — 1

C15. A drone hovers $75\sqrt{3}$ m above a straight road. It sees a car coming towards the point directly below it at an angle of depression of 30°. Ten seconds later the angle of depression is 60°. Find the speed of the car in m/s and in km/h. **[5]**



Correct figure — 1

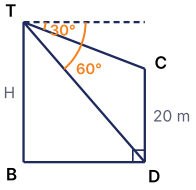
$BQ = \dfrac{75\sqrt{3}}{\tan 30^\circ} = 225$ m — 1

$BP = \dfrac{75\sqrt{3}}{\tan 60^\circ} = 75$ m — 1

Distance covered $d = 225 - 75 = 150$ m in 10 s — 1

Speed $= 15$ m/s $= 15 \times \dfrac{18}{5} = 54$ km/h — 1

C16. From the top of a tower, the angles of depression of the top and the foot of a 20 m high building are 30° and 60°. Find the height of the tower. **[3]**



$\tan 60^\circ = \dfrac{H}{d} \to d = \dfrac{H}{\sqrt{3}}$ — 1

$\tan 30^\circ = \dfrac{H-20}{d} \to H - 20 = \dfrac{d}{\sqrt{3}} = \dfrac{H}{3}$ — 1

$\dfrac{2H}{3} = 20 \to H = 30$ m — 1

C17. From the top of a 30 m tower, the angle of depression of a car is 30°. A student wrote: **[3]**
Step 1: the angle inside the triangle at the top of the tower is 30°.
Step 2: $\tan 30^\circ = \dfrac{d}{30}$. Step 3: $d = 10\sqrt{3}$ m.
Find the wrong step and the correct distance of the car from the foot of the tower.

Step 1 is wrong: 30° is measured from the horizontal, so it equals the angle at the car (alternate angles) — 1

$\tan 30^\circ = \dfrac{30}{d}$ — 1

$d = 30\sqrt{3} \approx 51.96$ m — 1

D

PART D · SOLUTIONS

Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Fire-Brigade Ladder [4]

(i) If $\theta = 60^\circ$, how high above the ground does the top of the ladder reach? **[1]**

$\text{Height} = 2 + 20\sin 60^\circ = 2 + 10\sqrt{3} \approx 19.32 \text{ m}$ 1

(ii) For $\theta = 60^\circ$, find the horizontal distance from the lower end of the ladder to the wall. **[1]**

$20\cos 60^\circ = 10 \text{ m}$ 1

(iii) A person waits at a window 12 m above the ground. At what angle θ must the ladder be set to just reach the window? **[2]**

$2 + 20\sin\theta = 12 \rightarrow \sin\theta = \frac{1}{2}$ 1

$\theta = 30^\circ$ 1

OR

(iii) If $\theta = 45^\circ$, find the height reached by the top of the ladder and the horizontal distance to the wall. **[2]**

$\text{Height} = 2 + 20\sin 45^\circ = 2 + 10\sqrt{2} \approx 16.14 \text{ m}$ 1

$\text{Horizontal distance} = 20\cos 45^\circ = 10\sqrt{2} \approx 14.14 \text{ m}$ 1

D2. Case Study · Coastguard Lighthouse [4]

(i) How far is boat P from the foot of the lighthouse? **[1]**

$BP = \frac{50}{\tan 60^\circ} = \frac{50\sqrt{3}}{3} \approx 28.87 \text{ m}$ 1

(ii) How far is boat Q from the foot of the lighthouse? **[1]**

$BQ = \frac{50}{\tan 30^\circ} = 50\sqrt{3} \approx 86.6 \text{ m}$ 1

(iii) Find the distance between the two boats. **[2]**

$PQ = 50\sqrt{3} - \frac{50}{\sqrt{3}} = \frac{100}{\sqrt{3}}$ 1

$= \frac{100\sqrt{3}}{3} \approx 57.74 \text{ m}$ 1

OR

(iii) Boat Q sails straight towards the lighthouse at 2 m/s. How long does it take to reach the position of boat P ? **[2]**

$PQ = \frac{100\sqrt{3}}{3} \approx 57.74 \text{ m}$ 1

$\text{Time} = \frac{57.74}{2} \approx 28.87 \text{ s}$ 1

D3. Case Study · Tower on a Rooftop [4]

(i) Find the height of the building. **[1]**

$BT = 30\tan 30^\circ = 10\sqrt{3} \approx 17.32 \text{ m}$ 1

(ii) Find the height of the top of the tower above the ground. **[1]**

$BF = 30\tan 60^\circ = 30\sqrt{3} \approx 51.96$ m — 1

(iii) Find the height of the tower. **[2]**

Tower $= 30\sqrt{3} - 10\sqrt{3}$ — 1

$= 20\sqrt{3} \approx 34.64$ m — 1

OR

(iii) Find the length of the line of sight from P to the top of the tower, and from P to the top of the building. **[2]**

$PF = \dfrac{30}{\cos 60^\circ} = 60$ m — 1

$PT = \dfrac{30}{\cos 30^\circ} = 20\sqrt{3} \approx 34.64$ m — 1

D4. Case Study · Hot-Air Balloon [4]

(i) Find the height of the balloon at B_1. **[1]**

$AB_1 = 120\tan 30^\circ = 40\sqrt{3} \approx 69.28$ m — 1

(ii) Find the height of the balloon at B_2. **[1]**

$AB_2 = 120\tan 60^\circ = 120\sqrt{3} \approx 207.85$ m — 1

(iii) The balloon took 4 minutes to rise from B_1 to B_2. Find its speed of rising in m/min. **[2]**

Rise $= 120\sqrt{3} - 40\sqrt{3} = 80\sqrt{3}$ m — 1

Speed $= \dfrac{80\sqrt{3}}{4} = 20\sqrt{3} \approx 34.64$ m/min — 1

OR

(iii) At B_2 the pilot looks down at O. Find the angle of depression of O and the distance OB_2. **[2]**

Angle of depression $= 60^\circ$ (alternate angles) — 1

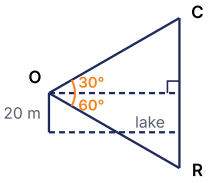
$OB_2 = \dfrac{120}{\cos 60^\circ} = 240$ m — 1

PART E · SOLUTIONS

E Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. From a point 20 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake. **[5]**



Correct figure: cloud C, its reflection R, eye O 20 m above the lake — 1

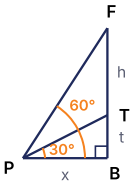
Let the cloud be H m above the lake and x m away: $\tan 30^\circ = \dfrac{H-20}{x}$ — 1

The reflection is H m below the surface: $\tan 60^\circ = \dfrac{H+20}{x}$ — 1

Divide: $\dfrac{H+20}{H-20} = \dfrac{\sqrt{3}}{1/\sqrt{3}} = 3$ — 1

$H + 20 = 3H - 60 \rightarrow H = 40$ m — 1

E2. A tower is topped by a flagstaff of height h. From a point on the ground, the angles of elevation of the bottom and the top of the flagstaff are 30° and 60°. Prove that the height of the tower is $\dfrac{h}{2}$. **[3]**



$\tan 30^\circ = \dfrac{t}{x} \rightarrow x = \sqrt{3}t$ — 1

$\tan 60^\circ = \dfrac{t+h}{x} \rightarrow t + h = \sqrt{3} \cdot \sqrt{3}t = 3t$ — 1

$2t = h \rightarrow t = \dfrac{h}{2}$ — 1

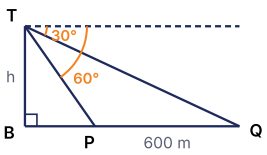
E3. The shadow of a tower grows 18 m longer when the Sun's elevation falls from 45° to 30°. Find the height of the tower. **[3]**

Shadows: h at 45° and $\sqrt{3}h$ at 30° — 1

$\sqrt{3}h - h = 18 \rightarrow h = \dfrac{18}{\sqrt{3}-1}$ — 1

$h = 9(\sqrt{3}+1) \approx 24.59$ m — 1

E4. From the top of a hill, the angles of depression of two milestones 600 m apart, on the same side and in line with the foot of the hill, are 60° and 30°. Find the height of the hill. **[3]**

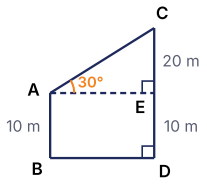


Nearer stone $BP = \dfrac{h}{\sqrt{3}}$, farther stone $BQ = \sqrt{3}h$ — 1

$\sqrt{3}h - \dfrac{h}{\sqrt{3}} = 600 \rightarrow \dfrac{2h}{\sqrt{3}} = 600$ — 1

$h = 300\sqrt{3} \approx 519.62$ m — 1

E5. Two poles 10 m and 30 m high stand on level ground. From the top of the shorter pole, the angle of elevation of the top of the taller pole is 30°. Find the distance between the poles and the length of a wire stretched between their tops. **[3]**

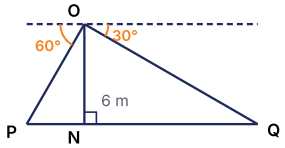


Difference in heights $CE = 30 - 10 = 20$ m — 1

$\tan 30^\circ = \dfrac{20}{AE} \rightarrow AE = 20\sqrt{3} \approx 34.64$ m — 1

Wire $AC = \dfrac{20}{\sin 30^\circ} = 40$ m — 1

E6. From a point O on a bridge, 6 m above the water, the angles of depression of the two banks of the river, on opposite sides, are 60° and 30°. Find the width of the river. **[3]**



$NP = \dfrac{6}{\tan 60^\circ} = 2\sqrt{3}$ m — 1

$NQ = \dfrac{6}{\tan 30^\circ} = 6\sqrt{3}$ m — 1

Width $= 8\sqrt{3} \approx 13.86$ m — 1