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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 14

Probability

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right answer without the listed outcomes can lose step marks.
4. Write the fraction in lowest terms and check that it lies between 0 and 1.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN PROBABILITY ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **2 marks, one part:** total outcomes ½ · favourable outcomes listed ½ · probability 1.
3. **2 or 3 marks, several parts:** each part carries its own mark: favourable outcomes ½ + probability ½ for a 1-mark part.
4. **Case study (4 marks):** 1 + 1 + 2. In the 2-mark part, each answer is worth 1 mark.
5. An unreduced but correct fraction (e.g. $\frac{6}{36}$) usually keeps the mark, but always reduce it.
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Basic ideas and rules

A1. Fill in the blanks: **(i)** The probability of an event that is certain to happen is ____. Such an event is called a ____ event. **(ii)** The probability of an event that cannot happen is ____. Such an event is called an ____ event. **(iii)** $P(E) + P(\text{not } E) =$ ____. **(iv)** The sum of the probabilities of all the elementary events of an experiment is ____. **(v)** The probability of any event is greater than or equal to ____ and less than or equal to ____. **[5]**

(i) 1; it is called a **sure** (certain) event. 1

(ii) 0; it is called an **impossible** event. 1

(iii) $P(E) + P(\text{not } E) = 1$. 1

(iv) 1. 1

(v) 0 and 1, i.e. $0 \le P(E) \le 1$. 1

A2. Which of these experiments have equally likely outcomes? Give a reason in each case. **(i)** A school team plays a football match. Result: it wins or it loses. **(ii)** A fair coin is tossed. Result: head or tail. **(iii)** A student who has not studied guesses the answer to a true–false question. Result: right or wrong. **(iv)** A bus is expected at 8 a.m. Result: it arrives on time or it is late. **[2]**

(i) **Not** equally likely: the result depends on the skill and form of the two teams. ½

(ii) **Equally likely**: a fair coin has the same chance of head and tail. ½

(iii) **Equally likely**: a pure guess between two choices gives each the same chance. ½

(iv) **Not** equally likely: traffic, weather and the driver decide whether the bus is late. ½

A3. Why is tossing a coin taken as a fair way of deciding which team bats first in a cricket match? **[1]**

Head and tail are equally likely, so each captain has the same chance, $\frac{1}{2}$, of winning the toss. No team is favoured. 1

Q	Answer	Working / reason
A4	**(b)** -0.4	A probability is never negative; it always lies between 0 and 1.

A5. If $P(E) = 0.07$, what is the probability of "not E"? [1]

$P(\text{not } E) = 1 - P(E) = 1 - 0.07 = 0.93$ — 1

A6. A jar contains only mint-flavoured toffees. Ravi takes out one toffee without looking. Find the probability that it is: **(i)** orange-flavoured **(ii)** mint-flavoured [2]

(i) There is no orange toffee, so this is an impossible event: $P = 0$. — 1

(ii) Every toffee is mint-flavoured, so this is a sure event: $P = 1$. — 1

A7. The probability that two students chosen at random from a class do *not* have the same birthday is 0.994. What is the probability that they have the same birthday? [1]

"Same birthday" is the complement of "not the same birthday": $P = 1 - 0.994 = 0.006$ — 1

Balls, coins, fish and other everyday situations

A8. A bag contains 4 red, 6 white and 5 green balls. One ball is taken out at random. Find the probability that it is: **(i)** white **(ii)** not green **(iii)** neither red nor white [3]

Total balls $= 4 + 6 + 5 = 15$

(i) White balls $\to 6$; $P = \dfrac{6}{15} = \dfrac{2}{5}$ — ½ + ½

(ii) Not green = red or white: $4 + 6 \to 10$; $P = \dfrac{10}{15} = \dfrac{2}{3}$ — ½ + ½

(iii) Neither red nor white = green $\to 5$; $P = \dfrac{5}{15} = \dfrac{1}{3}$ — ½ + ½

A9. A money box contains sixty ₹1 coins, forty ₹2 coins, twenty-five ₹5 coins and fifteen ₹10 coins. When the box is turned upside down, one coin falls out at random. Find the probability that the coin: **(i)** is a ₹5 coin **(ii)** is not a ₹10 coin [2]

Total coins $= 60 + 40 + 25 + 15 = 140$

(i) ₹5 coins $\to 25$; $P = \dfrac{25}{140} = \dfrac{5}{28}$ — ½ + ½

(ii) Not a ₹10 coin: $140 - 15 \to 125$; $P = \dfrac{125}{140} = \dfrac{25}{28}$ — ½ + ½

A10. A fish tank has 6 male fish and 9 female fish. One fish is taken out at random. What is the probability that it is a male fish? [1]

Total fish $= 6 + 9 = 15$, male fish $= 6$ — ½

$P(\text{male}) = \dfrac{6}{15} = \dfrac{2}{5}$ — ½

A11. In a game, an arrow is spun and comes to rest pointing at one of the numbers 1 to 10 (see the figure). All the outcomes are equally likely. Find the probability that it points at: **(i)** an odd number **(ii)** a number greater than 3 **(iii)** a prime number **[3]**

Total outcomes $= 10$

(i) Odd numbers: 1, 3, 5, 7, 9 → 5; $P = \frac{5}{10} = \frac{1}{2}$ — ½ + ½

(ii) Greater than 3: 4, 5, …, 10 → 7; $P = \frac{7}{10}$ — ½ + ½

(iii) Primes: 2, 3, 5, 7 → 4; $P = \frac{4}{10} = \frac{2}{5}$ — ½ + ½

A12. A die is thrown once. Find the probability of getting: **(i)** a composite number **(ii)** a number lying between 1 and 5 **(iii)** an even prime number **[3]**

Total outcomes $= 6$

(i) Composite numbers: 4, 6 → 2; $P = \frac{2}{6} = \frac{1}{3}$ — ½ + ½

(ii) Between 1 and 5: 2, 3, 4 → 3; $P = \frac{3}{6} = \frac{1}{2}$ — ½ + ½

(iii) Even prime: 2 → 1; $P = \frac{1}{6}$ — ½ + ½

Playing cards

A13. One card is drawn from a well-shuffled pack of 52 cards. Find the probability of getting: **(i)** a black queen **(ii)** a face card **(iii)** a black face card **(iv)** the 7 of clubs **(v)** a heart **(vi)** a card that is not an ace **[3]**

Total outcomes $= 52$

(i) Black queens: Q♠, Q♣ → 2; $P = \frac{2}{52} = \frac{1}{26}$ — ½

(ii) Face cards: 4 kings, 4 queens, 4 jacks → 12; $P = \frac{12}{52} = \frac{3}{13}$ — ½

(iii) Black face cards: 3 spades + 3 clubs → 6; $P = \frac{6}{52} = \frac{3}{26}$ — ½

(iv) Only one 7 of clubs → 1; $P = \frac{1}{52}$ — ½

(v) Hearts → 13; $P = \frac{13}{52} = \frac{1}{4}$ — ½

(vi) Not an ace: $52 - 4 = 48$; $P = \frac{48}{52} = \frac{12}{13}$ — ½

A14. Six cards – the 9, 10, jack, queen, king and ace of hearts – are shuffled and placed face down. One card is picked at random. **(i)** What is the probability that it is the king? **(ii)** If the king is drawn first and put aside, what is the probability that the second card picked is (a) an ace (b) a king? **[3]**

(i) One king among 6 cards: $P(\text{king}) = \frac{1}{6}$ — 1

(ii) The king is put aside, so 5 cards are left.

(a) One ace among 5: $P(\text{ace}) = \frac{1}{5}$ — 1

(b) No king is left: $P(\text{king}) = \frac{0}{5} = 0$ (impossible event) — 1

Defective items, discs, letters and lotteries

A15. 15 faulty bulbs have been mixed by mistake with 105 good ones. One bulb is taken out at random. Find the probability that it is good. [1]

Total bulbs $= 15 + 105 = 120$ ½

$P(\text{good}) = \dfrac{105}{120} = \dfrac{7}{8}$ ½

A16. A carton of 25 eggs has 5 rotten eggs. **(i)** One egg is taken out at random. What is the probability that it is rotten? **(ii)** The egg taken out in (i) is rotten and is not put back. One more egg is taken out. What is the probability that it is not rotten? [2]

(i) $P(\text{rotten}) = \dfrac{5}{25} = \dfrac{1}{5}$ 1

(ii) Eggs left $= 24$, good eggs $= 20$: $P(\text{not rotten}) = \dfrac{20}{24} = \dfrac{5}{6}$ 1

A17. A box contains 80 discs numbered 1 to 80. One disc is taken out at random. Find the probability that it bears: **(i)** a two-digit number **(ii)** a perfect square number **(iii)** a number divisible by 6 [3]

Total discs $= 80$

(i) Two-digit numbers 10 to 80: $80 - 10 + 1 \to 71$; $P = \dfrac{71}{80}$ ½ + ½

(ii) Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64 $\to 8$; $P = \dfrac{8}{80} = \dfrac{1}{10}$ ½ + ½

(iii) Multiples of 6: 6, 12, …, 78 $\to 13$; $P = \dfrac{13}{80}$ ½ + ½

A18. The six faces of a cube carry the letters P, Q, R, P, S, P. The cube is thrown once. Find the probability of getting: **(i)** the letter P **(ii)** the letter Q [2]

Total outcomes $= 6$

(i) Faces with P $\to 3$; $P = \dfrac{3}{6} = \dfrac{1}{2}$ ½ + ½

(ii) Faces with Q $\to 1$; $P = \dfrac{1}{6}$ ½ + ½

A19. A school fair has 500 scratch cards: 5 win a bicycle, 25 win a book, 70 win a pen and the rest win nothing. One card is picked at random. Find the probability that it: **(i)** wins a book **(ii)** wins nothing [2]

Total cards $= 500$; **prize cards** $= 5 + 25 + 70 = 100$

(i) Book cards $\to 25$; $P = \dfrac{25}{500} = \dfrac{1}{20}$ ½ + ½

(ii) Cards with nothing: $500 - 100 \to 400$; $P = \dfrac{400}{500} = \dfrac{4}{5}$ ½ + ½

A20. A jar has 36 marbles. Some are red and the rest are yellow. The probability of drawing a red marble is $\dfrac{5}{9}$. How many yellow marbles are there? [2]

Red marbles $= \dfrac{5}{9} \times 36 = 20$ 1

Yellow marbles $= 36 - 20 = 16$ 1

Coins and two dice

A21. Two dice, one red and one blue, are thrown at the same time. **(i)** How many outcomes are possible? **(ii)** Find the probability of each possible sum 2, 3, …, 12. **(iii)** A student says: "There are 11 possible sums, so each has probability $\frac{1}{11}$." Do you agree? Give a reason. **[3]**

(i) Each die has 6 outcomes, so there are $6 \times 6 = 36$ ordered pairs (red, blue). ½

(ii) Sum 2: $\frac{1}{36}$ · 3: $\frac{2}{36} = \frac{1}{18}$ · 4: $\frac{3}{36} = \frac{1}{12}$ · 5: $\frac{4}{36} = \frac{1}{9}$ · 6: $\frac{5}{36}$ · 7: $\frac{6}{36} = \frac{1}{6}$ · 8: $\frac{5}{36}$ · 9: $\frac{1}{9}$ · 10: $\frac{1}{12}$ · 11: $\frac{1}{18}$ · 12: $\frac{1}{36}$ 1½

(iii) No. The 11 sums are **not equally likely**: sum 7 comes in 6 ways but sum 2 in only 1 way. So each sum does not have probability $\frac{1}{11}$. 1

A22. A coin is tossed three times. Meera wins if she gets exactly two heads; otherwise she loses. Find the probability that Meera wins and the probability that she loses. **[2]**

Outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT → 8 ½

Exactly two heads: HHT, HTH, THH → 3 ½

$P(\text{win}) = \dfrac{3}{8}$ ½

$P(\text{lose}) = 1 - \dfrac{3}{8} = \dfrac{5}{8}$ ½

A23. A die is thrown twice. What is the probability that: **(i)** 3 will not come up either time **(ii)** 3 will come up at least once **[2]**

Total outcomes = 36

(i) No 3 on either throw: $5 \times 5 = 25$ outcomes; $P = \dfrac{25}{36}$ 1

(ii) At least one 3 $= 1 - \dfrac{25}{36} = \dfrac{11}{36}$ 1

A24. When two coins are tossed, Rahul says: "The probability of getting one head and one tail is $\frac{1}{3}$, because the three possible results are two heads, two tails, or one of each." Is he right? Give a reason. **[2]**

No, Rahul is not right. ½

The equally likely outcomes are HH, HT, TH, TT → 4 (HT and TH are different). ½

One head and one tail: HT, TH → 2 ½

$P = \dfrac{2}{4} = \dfrac{1}{2}$, not $\frac{1}{3}$. ½

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	(a) $\frac{18}{23}$	$\frac{18}{23}$ lies between 0 and 1. The others are above 1 or negative.
B2	(b) $\frac{7}{11}$	11 letters: M (1), S (4), P (2) are consonants → 7; I (4) is the vowel. $P = \frac{7}{11}$
B3	(c) $\frac{6}{13}$	Red cards 26 + black queens 2 = 28 are red or a queen. Neither $= 52 - 28 = 24$; $\frac{24}{52} = \frac{6}{13}$
B4	(d) $\frac{1}{4}$	There are 25 primes from 1 to 100: $\frac{25}{100} = \frac{1}{4}$
B5	(b) $\frac{7}{36}$	Sum 5: (1,4), (2,3), (3,2), (4,1); sum 10: (4,6), (5,5), (6,4) → 7; $P = \frac{7}{36}$
B6	(c) $\frac{2}{5}$	Multiples of 3: 3, 6, 9, 12, 15, 18; of 7: 7, 14 → 8 (none common); $\frac{8}{20} = \frac{2}{5}$
B7	(a) 480	$\frac{x}{6000} = 0.08 \Rightarrow x = 480$
B8	(b) $\frac{2}{3}$	Not black $= 3 + 7 = 10$ of 15: $\frac{10}{15} = \frac{2}{3}$
B9	(d) $\frac{1}{7}$	365 days = 52 weeks + 1 day; that day is a Sunday in 1 case of 7.
B10	(c) $\frac{5}{7}$	$x^2 \le 4$ for $x = -2, -1, 0, 1, 2$ → 5 of 7.

Short answer with reasons

B11. Sara says: "The probability of a head is $\frac{1}{2}$, so if I toss a coin 10 times I will surely get exactly 5 heads." Is she right? Give a reason. **[2]**

No, she is not right. — 1

$P(H) = \frac{1}{2}$ only means that a head and a tail are equally likely on each toss. In 10 tosses she may get 3, 6 or any number of heads; the probability tells us what to expect over a very large number of tosses. — 1

B12. A die is thrown once. Is the probability of getting a number less than 3 equal to the probability of getting a number greater than 3? Give a reason. **[2]**

Less than 3: 1, 2 → $P = \frac{2}{6} = \frac{1}{3}$ — 1

Greater than 3: 4, 5, 6 → $P = \frac{3}{6} = \frac{1}{2}$ — ½

$\frac{1}{3} \neq \frac{1}{2}$, so the two probabilities are **not** equal. — ½

B13. Two coins are tossed together. Is the probability of getting at least one head equal to the probability of getting at most one head? Justify. [2]

Outcomes: HH, HT, TH, TT → 4 ½

At least one head: HH, HT, TH → $P = \frac{3}{4}$ ½

At most one head: HT, TH, TT → $P = \frac{3}{4}$ ½

Yes, the two probabilities are equal. ½

B14. A family has three children. Assuming a boy and a girl are equally likely, find the probability that the family has at least one girl. [2]

Outcomes: BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG → 8 ½

No girl: BBB only → 1 ½

$P(\text{at least one girl}) = 1 - \frac{1}{8} = \frac{7}{8}$ 1

B15. A die is thrown once. Find the probability of getting a number which is: **(i)** a multiple of both 2 and 3 **(ii)** a number whose square is less than 20 [2]

Total outcomes = 6

(i) Multiple of both 2 and 3 = multiple of 6: 6 → 1; $P = \frac{1}{6}$ ½ + ½

(ii) Square less than 20: $1^2, 2^2, 3^2, 4^2$ → numbers 1, 2, 3, 4 → 4; $P = \frac{4}{6} = \frac{2}{3}$ ½ + ½

Short answer

B16. Two dice are thrown at the same time. Find the probability that: **(i)** the product of the numbers is 12 **(ii)** the difference of the numbers is 3 **(iii)** the number on the first die is greater than the number on the second [3]

Total outcomes = 36

(i) Product 12: (2,6), (6,2), (3,4), (4,3) → 4; $P = \frac{4}{36} = \frac{1}{9}$ ½ + ½

(ii) Difference 3: (1,4), (4,1), (2,5), (5,2), (3,6), (6,3) → 6; $P = \frac{6}{36} = \frac{1}{6}$ ½ + ½

(iii) First greater than second: 1 + 2 + 3 + 4 + 5 (for first = 2, 3, 4, 5, 6) → 15; $P = \frac{15}{36} = \frac{5}{12}$ ½ + ½

B17. A box contains cards bearing the odd numbers 3, 5, 7, 9, …, 35, one number on each card. A card is drawn at random. Find the probability that the number on it is: **(i)** a prime number **(ii)** divisible by 3 **(iii)** a perfect square [3]

Number of cards = $\frac{35-3}{2} + 1 = 17$

(i) Primes: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 → 10; $P = \frac{10}{17}$ ½ + ½

(ii) Divisible by 3: 3, 9, 15, 21, 27, 33 → 6; $P = \frac{6}{17}$ ½ + ½

(iii) Perfect squares: 9, 25 → 2; $P = \frac{2}{17}$ ½ + ½

B18. A bag contains 24 balls: x are red, $2x$ are white and $3x$ are blue. A ball is drawn at random. Find the probability that it is: **(i)** not red **(ii)** white [3]

$x + 2x + 3x = 24 \Rightarrow 6x = 24 \Rightarrow x = 4$. Red 4, white 8, blue 12. 1

(i) Not red $= 8 + 12 = 20$: $P = \dfrac{20}{24} = \dfrac{5}{6}$ 1

(ii) White: $P = \dfrac{8}{24} = \dfrac{1}{3}$ 1

PART C · SOLUTIONS

C Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Two dice

Q	Answer	Working / reason
C1	(b) $\frac{1}{6}$	6 doublets out of 36: $\frac{6}{36} = \frac{1}{6}$
C2	(b) $\frac{5}{36}$	(2,6), (3,5), (4,4), (5,3), (6,2) → 5 of 36
C3	(d) $\frac{1}{4}$	Both numbers odd: $3 \times 3 = 9$ of 36 $= \frac{1}{4}$
C4	(a) $\frac{1}{12}$	Sum 2: (1,1); sum 3: (1,2), (2,1) → 3 of 36 $= \frac{1}{12}$
C5	(c) $\frac{1}{9}$	(5,5), (5,6), (6,5), (6,6) → 4 of 36 $= \frac{1}{9}$
C6	(b)	A is true (6 of 36). R is true, but it does not explain A: the 11 sums are not equally likely.

C7. Two dice are thrown at the same time. Find the probability that the product of the numbers is a perfect square. [2]

Total outcomes $= 36$ ½

Product 1: (1,1) · 4: (1,4), (4,1), (2,2) · 9: (3,3) · 16: (4,4) · 25: (5,5) · 36: (6,6) → 8 ½

$P = \dfrac{8}{36} = \dfrac{2}{9}$ 1

C8. Two dice are thrown at the same time. Find the probability that the sum of the numbers is a perfect square. [2]

Total outcomes $= 36$ ½

Sum 4: (1,3), (2,2), (3,1) · sum 9: (3,6), (4,5), (5,4), (6,3) → 7 ½

$P = \dfrac{7}{36}$ 1

C9. Two dice of different colours are thrown together. Write the number of possible outcomes. Find the probability of getting: **(i)** the same number on both dice **(ii)** different numbers on the two dice **(iii)** an odd sum **[3]**

Total outcomes $= 6 \times 6 = 36$

(i) Doublets: (1,1), (2,2), …, (6,6) → 6; $P = \frac{6}{36} = \frac{1}{6}$ ½ + ½

(ii) Different numbers: $36 - 6 \to 30$; $P = \frac{30}{36} = \frac{5}{6}$ ½ + ½

(iii) Odd sum (one odd, one even): $3 \times 3 + 3 \times 3 \to 18$; $P = \frac{18}{36} = \frac{1}{2}$ ½ + ½

C10. Two dice are thrown at the same time. Find the probability that: **(i)** the sum is less than 5 **(ii)** the sum is a multiple of 4 **(iii)** a doublet of odd numbers appears **[3]**

Total outcomes $= 36$

(i) Sum less than 5 (2, 3, 4): $1 + 2 + 3 \to 6$; $P = \frac{6}{36} = \frac{1}{6}$ ½ + ½

(ii) Sum a multiple of 4 (4, 8, 12): $3 + 5 + 1 \to 9$; $P = \frac{9}{36} = \frac{1}{4}$ ½ + ½

(iii) Doublet of odd numbers: (1,1), (3,3), (5,5) → 3; $P = \frac{3}{36} = \frac{1}{12}$ ½ + ½

C11. A die is thrown twice. Find the probability that: **(i)** the first throw shows a prime number and the second an even number **(ii)** at least one throw shows 6 **(iii)** the sum of the two numbers is 9 or 11 **[3]**

Total outcomes $= 36$

(i) First throw prime (2, 3, 5) and second even (2, 4, 6): $3 \times 3 \to 9$; $P = \frac{9}{36} = \frac{1}{4}$ ½ + ½

(ii) At least one 6: $36 - 5 \times 5 \to 11$; $P = \frac{11}{36}$ ½ + ½

(iii) Sum 9: 4 outcomes; sum 11: (5,6), (6,5) → $4 + 2 \to 6$; $P = \frac{6}{36} = \frac{1}{6}$ ½ + ½

C12. Two dice are thrown together. Find the probability that the difference of the numbers on them is: **(i)** 2 **(ii)** 0 **(iii)** at most 1 **[3]**

Total outcomes $= 36$

(i) Difference 2: (1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4) → 8; $P = \frac{8}{36} = \frac{2}{9}$ ½ + ½

(ii) Difference 0: the 6 doublets → 6; $P = \frac{6}{36} = \frac{1}{6}$ ½ + ½

(iii) Difference 0 or 1: 6 doublets + 10 pairs like (1,2), (2,1), …, (5,6), (6,5) → 16; $P = \frac{16}{36} = \frac{4}{9}$ ½ + ½

Numbered cards, tickets and discs

Q	Answer	Working / reason
C13	(b) $\frac{6}{25}$	4, 8, …, 48 → 12 of 50 = $\frac{6}{25}$
C14	(c) $\frac{2}{25}$	50 cards; squares 16, 25, 36, 49 → 4; $\frac{4}{50} = \frac{2}{25}$
C15	(a) $\frac{1}{8}$	80 cards. Squares 4, 9, …, 81 → 8; cubes 8, 27, 64 → 3; 64 is in both → $8 + 3 - 1 = 10$; $\frac{10}{80} = \frac{1}{8}$
C16	(b) $\frac{1}{5}$	7, 11, 13 → 3 of 15 = $\frac{1}{5}$

C17. Cards numbered 7 to 106 are placed in a box. One card is drawn at random. Find the probability that the number on the card is: **(i)** a perfect square **(ii)** a two-digit number [2]

Number of cards $= 106 - 7 + 1 = 100$

(i) Perfect squares: 9, 16, 25, 36, 49, 64, 81, 100 → 8; $P = \frac{8}{100} = \frac{2}{25}$ ½ + ½

(ii) Two-digit numbers 10 to 99 → 90; $P = \frac{90}{100} = \frac{9}{10}$ ½ + ½

C18. Thirty cards numbered 1 to 30 are shuffled. One card is drawn at random. Find the probability that its number is: **(i)** divisible by both 2 and 3 **(ii)** a prime number greater than 10 [2]

Total cards $= 30$

(i) Divisible by 2 and 3 (by 6): 6, 12, 18, 24, 30 → 5; $P = \frac{5}{30} = \frac{1}{6}$ ½ + ½

(ii) Primes greater than 10: 11, 13, 17, 19, 23, 29 → 6; $P = \frac{6}{30} = \frac{1}{5}$ ½ + ½

C19. A box contains 100 cards numbered 1 to 100. One card is drawn at random. Find the probability that its number is: **(i)** even but not divisible by 4 **(ii)** a perfect cube **(iii)** divisible by 7 [3]

Total cards $= 100$

(i) Even but not divisible by 4: 2, 6, 10, …, 98 → 25; $P = \frac{25}{100} = \frac{1}{4}$ ½ + ½

(ii) Perfect cubes: 1, 8, 27, 64 → 4; $P = \frac{4}{100} = \frac{1}{25}$ ½ + ½

(iii) Divisible by 7: 7, 14, …, 98 → 14; $P = \frac{14}{100} = \frac{7}{50}$ ½ + ½

C20. Tickets bearing the even numbers 2, 4, 6, …, 50 are put in a bag. One ticket is drawn at random. Find the probability that its number is: **(i)** a multiple of 4 **(ii)** a number ending in 0 **(iii)** a perfect square [3]

Number of tickets $= 25$

(i) Multiples of 4: 4, 8, …, 48 → 12; $P = \frac{12}{25}$ ½ + ½

(ii) Ending in 0: 10, 20, 30, 40, 50 → 5; $P = \frac{5}{25} = \frac{1}{5}$ ½ + ½

(iii) Perfect squares: 4, 16, 36 → 3; $P = \frac{3}{25}$ ½ + ½

Rules: complement, sure and impossible events

Q	Answer	Working / reason
C21	**(c)** 0.63	$1 - 0.37 = 0.63$
C22	**(a)** 1 – p	$P(E) + P(\text{not } E) = 1$
C23	**(c)** 3	$\frac{x}{12} = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow x = 3$
C24	**(b)** Getting a number less than 7 on a die	Every face 1–6 is less than 7, so $P = 1$.
C25	**(a)**	All 25 numbers are natural numbers, so the event is sure; R explains A.
C26	**(d)**	A is false: 4 is in neither event and $\frac{3}{6} + \frac{2}{6} = \frac{5}{6} \neq 1$. R is true.
C27	**(a)**	$P(\text{four}) = \frac{12}{60} = \frac{1}{5}$, so $P(\text{not}) = 1 - \frac{1}{5} = \frac{4}{5}$; R explains A.

Cards from a pack

Q	Answer	Working / reason
C28	(b) $\frac{3}{26}$	6 red face cards: $\frac{6}{52} = \frac{3}{26}$
C29	(c) $\frac{9}{13}$	Hearts 13 + other kings 3 = 16; neither = 36; $\frac{36}{52} = \frac{9}{13}$
C30	(a) $\frac{13}{44}$	Cards left = 44; all 13 diamonds are there: $\frac{13}{44}$
C31	(d) $\frac{1}{4}$	48 left; hearts left = 12; $\frac{12}{48} = \frac{1}{4}$

C32. One card is drawn from a well-shuffled pack of 52 cards. Find the probability of getting: **(i)** a black card that is not a king **(ii)** a red number card (2 to 10) [2]

Total cards = 52

(i) Black cards that are not kings: $26 - 2 \to 24$; $P = \frac{24}{52} = \frac{6}{13}$ ½ + ½

(ii) Red number cards 2 to 10: $9 \times 2 \to 18$; $P = \frac{18}{52} = \frac{9}{26}$ ½ + ½

C33. The jack, queen and king of spades are removed from a pack of 52 cards. One card is drawn from the remaining cards. Find the probability that it is: **(i)** a face card **(ii)** a spade **(iii)** a black card [3]

Cards left = $52 - 3 = 49$

(i) Face cards left: $12 - 3 \to 9$; $P = \frac{9}{49}$ ½ + ½

(ii) Spades left: $13 - 3 \to 10$; $P = \frac{10}{49}$ ½ + ½

(iii) Black cards left: $26 - 3 \to 23$; $P = \frac{23}{49}$ ½ + ½

C34. The two red aces and the two black queens are removed from a pack of 52 cards. One card is drawn from the rest. Find the probability that it is: **(i)** a red card **(ii)** a queen **(iii)** an ace or a king [3]

Cards left = $52 - 4 = 48$

(i) Red cards left: $26 - 2 \to 24$; $P = \frac{24}{48} = \frac{1}{2}$ ½ + ½

(ii) Queens left: the 2 red queens $\to 2$; $P = \frac{2}{48} = \frac{1}{24}$ ½ + ½

(iii) Aces or kings: 2 black aces + 4 kings $\to 6$; $P = \frac{6}{48} = \frac{1}{8}$ ½ + ½

Data and case-based probability

C35. In a survey of 200 families of a town, 64 families use only a car, 48 use only a scooter, 18 use both and the rest use neither. One family is chosen at random. Find the probability that the family: **(i)** uses a car **(ii)** uses neither a car nor a scooter [2]

Total families = 200

(i) Uses a car = only car + both: $64 + 18 \to 82$; $P = \frac{82}{200} = \frac{41}{100}$ ½ + ½

(ii) Uses neither: $200 - 64 - 48 - 18 \to 70$; $P = \frac{70}{200} = \frac{7}{20}$ ½ + ½

Q	Answer	Working / reason
C36	(b) 240	Defective = $0.04 \times 250 = 10$; not defective = 240

Coins

Q	Answer	Working / reason
C37	(d) $\frac{3}{4}$	HT, TH, TT → 3 of 4

C38. Three coins are tossed together. Find the probability of getting: **(i)** at least two tails **(ii)** no tail [2]

Outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT → 8

(i) At least two tails: HTT, THT, TTH, TTT → 4; $P = \frac{4}{8} = \frac{1}{2}$ ½ + ½

(ii) No tail: HHH → 1; $P = \frac{1}{8}$ ½ + ½

Q	Answer	Working / reason
C39	(a)	HT, TH → 2 of 4 = $\frac{1}{2}$; R explains A.

Calendar and letters

Q	Answer	Working / reason
C40	(b) $\frac{1}{7}$	One extra day; it is a Friday in 1 case of 7.
C41	(c)	A is true (2 of the 7 day-pairs include Tuesday). R is false: a non-leap year gives $\frac{1}{7}$.

C42. A letter is chosen at random from the letters of the word EXAMINATION. Find the probability that the letter is: **(i)** a vowel **(ii)** the letter N [2]

Letters: E, X, A, M, I, N, A, T, I, O, N → 11

(i) Vowels: E, A, I, A, I, O → 6; $P = \frac{6}{11}$ ½ + ½

(ii) The letter N appears → 2; $P = \frac{2}{11}$ ½ + ½

PART D · SOLUTIONS

D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Ludo Evening [4]

(i) What is the probability of getting a doublet? **[1]**

Total outcomes $= 36$; doublets $= 6$ ½

$P = \frac{6}{36} = \frac{1}{6}$ ½

(ii) What is the probability that the total is exactly 10? **[1]**

Total 10: (4,6), (5,5), (6,4) $\rightarrow 3$ ½

$P = \frac{3}{36} = \frac{1}{12}$ ½

(iii) Find the probability that a token can leave its home square on a turn, and the probability that it cannot. **[2]**

Total 10 or more: $3 + 2 + 1 = 6$ outcomes; $P = \frac{6}{36} = \frac{1}{6}$ 1

$P(\text{cannot}) = 1 - \frac{1}{6} = \frac{5}{6}$ 1

OR

(iii) Find the probability that the total on the two dice is a prime number. **[2]**

Prime totals 2, 3, 5, 7, 11: $1 + 2 + 4 + 6 + 2 = 15$ outcomes 1

$P = \frac{15}{36} = \frac{5}{12}$ 1

D2. Case Study · Spin the Wheel [4]

(i) Find the probability that the arrow stops at a multiple of 3. **[1]**

Multiples of 3: 3, 6, 9, 12 $\rightarrow 4$ ½

$P = \frac{4}{12} = \frac{1}{3}$ ½

(ii) Find the probability that the arrow stops at a factor of 12. **[1]**

Factors of 12: 1, 2, 3, 4, 6, 12 $\rightarrow 6$ ½

$P = \frac{6}{12} = \frac{1}{2}$ ½

(iii) Find the probability that a player wins a prize, and the probability that a player does not win. **[2]**

Primes: 2, 3, 5, 7, 11 $\rightarrow 5$; $P(\text{win}) = \frac{5}{12}$ 1

$P(\text{no win}) = 1 - \frac{5}{12} = \frac{7}{12}$ 1

OR

(iii) Find the probability that the arrow stops at an even prime or at a number greater than 9. **[2]**

Even prime: 2; greater than 9: 10, 11, 12 $\rightarrow 4$ numbers 1

$P = \frac{4}{12} = \frac{1}{3}$ 1

D3. Case Study · Toffee Jar [4]

(i) What is the probability of picking a mango toffee? **[1]**

$P(\text{mango}) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$ 1

(ii) How many toffees are there in the jar? **[1]**

$\frac{15}{\text{total}} = \frac{1}{2} \Rightarrow \text{total} = 30$ 1

(iii) How many orange toffees and how many lime toffees are there? **[2]**

$\text{Orange} = \frac{1}{3} \times 30 = 10$ 1

$\text{Lime} = \frac{1}{6} \times 30 = 5$ 1

OR

(iii) If 6 more lime toffees are put into the jar, find the probability of picking a lime toffee. **[2]**

$\text{Lime} = 5 + 6 = 11$, $\text{total} = 30 + 6 = 36$ 1

$P(\text{lime}) = \frac{11}{36}$ 1

D4. Case Study · Blood Donation Camp [4]

(i) Find the probability that the donor has blood group AB+. **[1]**

$P(\text{AB+}) = \frac{5}{50} = \frac{1}{10}$ 1

(ii) Find the probability that the donor does not have blood group O+. **[1]**

Not O+: $50 - 18 = 32$; $P = \frac{32}{50} = \frac{16}{25}$ 1

(iii) Find the probability that the donor has blood group A+ or B+, and the probability that the donor has neither A+ nor B+. **[2]**

A+ or B+: $12 + 15 = 27$; $P = \frac{27}{50}$ 1

Neither: $50 - 27 = 23$; $P = \frac{23}{50}$ 1

OR

(iii) If 10 more O+ donors join, find the new probability that the chosen donor has blood group O+. **[2]**

O+ $= 28$, total $= 60$ 1

$P(\text{O+}) = \frac{28}{60} = \frac{7}{15}$ 1

PART E · SOLUTIONS

E Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. Two dice are thrown together. Find the probability that the product of the numbers is a multiple of 3. **[3]**

Total outcomes $= 36$ ½

The product is **not** a multiple of 3 when neither die shows 3 or 6: each die has 4 such faces → $4 \times 4 = 16$ 1

Favourable $= 36 - 16 = 20$ ½

$P = \dfrac{20}{36} = \dfrac{5}{9}$ 1

E2. A die is thrown twice. Find the probability that the larger of the two numbers obtained is 4. **[3]**

Total outcomes $= 36$ ½

Both numbers ≤ 4: $4 \times 4 = 16$; both ≤ 3: $3 \times 3 = 9$ 1

Larger number exactly 4: $16 - 9 = 7$ → (1,4), (2,4), (3,4), (4,4), (4,1), (4,2), (4,3) ½

$P = \dfrac{7}{36}$ 1

E3. A number x is chosen at random from 1, 2, 3, 4 and a number y from 1, 4, 9, 16. Find the probability that $xy < 16$. **[3]**

Total pairs $= 4 \times 4 = 16$ ½

$x = 1$: 1, 4, 9 → 3; $x = 2$: 2, 8 → 2; $x = 3$: 3, 12 → 2; $x = 4$: 4 → 1 1

Favourable $= 3 + 2 + 2 + 1 = 8$ ½

$P = \dfrac{8}{16} = \dfrac{1}{2}$ 1

E4. Two friends are each asked to choose a number from 1 to 5 at random. Find the probability that they choose the same number. **[2]**

Total pairs $= 5 \times 5 = 25$ ½

Same number: (1,1), (2,2), …, (5,5) → 5 ½

$P = \dfrac{5}{25} = \dfrac{1}{5}$ 1

E5. A number is chosen at random from 1 to 100. Find the probability that it is neither a multiple of 2 nor a multiple of 5. **[3]**

Multiples of 2: 50; multiples of 5: 20 ½

Multiples of both (of 10): 10 ½

Multiples of 2 or 5: $50 + 20 - 10 = 60$ ½

Neither: $100 - 60 = 40$ ½

$P = \dfrac{40}{100} = \dfrac{2}{5}$ 1

E6. In a bag, red and green balls are in the ratio 3 : 5. When 8 more red balls are added, the ratio becomes 1 : 1. Find the number of balls in the bag at first, and the probability of drawing a red ball before and after adding the balls. **[3]**

Let red $= 3k$, green $= 5k$. Then $3k + 8 = 5k \Rightarrow k = 4$ — 1

At first: red 12, green 20, total 32 — ½

Before: $P(\text{red}) = \dfrac{12}{32} = \dfrac{3}{8}$ — ½

After: red 20, total 40: $P(\text{red}) = \dfrac{20}{40} = \dfrac{1}{2}$ — 1

E7. Each of two dice has the numbers 1, 2, 2, 3, 3, 3 on its six faces. Both are thrown together. Find the probability that the total score is: **(i)** 6 **(ii)** 2 **(iii)** 4 **[3]**

Treat the six faces as different: total outcomes $= 6 \times 6 = 36$ — ½

(i) Total 6 needs 3 and 3: $3 \times 3 = 9$; $P = \dfrac{9}{36} = \dfrac{1}{4}$ — 1

(ii) Total 2 needs 1 and 1: $1 \times 1 = 1$; $P = \dfrac{1}{36}$ — ½

(iii) Total 4: (1,3) 1×3 + (3,1) 3×1 + (2,2) $2 \times 2 = 10$; $P = \dfrac{10}{36} = \dfrac{5}{18}$ — 1