Affiliated to CBSE, New Delhi (Affiliation No. 1030642)

GNPS

GOMTI NANDAN PUBLIC SCHOOL

Gandhi Ward, Bina (M.P.) 470113. | 9165 609 333. | info@mygnps.com | www.mygnps.com

CLASS X · MATHEMATICS BASIC (241) · CBSE BOARD EXAMINATION 2027

CHAPTER 1 BASIC

# Real Numbers

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Basic booklet | Step marks shown line by line | MCQ keys with working

## INSIDE THIS BOOKLET


| | | |
|---|---|---|
| ✓ | **Marking Guide** How step marks are given | 2 |
| A | **Guided Practice – Solutions** 6 filled-in methods | 2 |
| B | **NCERT Practice – Solutions** 13 solutions | 3 |
| C | **Board Pattern Practice – Solutions** 31 solutions | 5 |
| D | **Case-Based Questions – Solutions** 3 case studies, all parts and ORs | 9 |


## HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check an HCF by dividing both numbers by it, and an LCM by dividing it by both numbers.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

# Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

### HOW MARKS ARE GIVEN IN REAL NUMBERS ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **$\sqrt{p}$ is irrational (3 marks):** assume $\frac{a}{b}$ with $a, b$ coprime ½ · $p$ divides $a^2$ so $p$ divides $a$ 1 · $p$ divides $b$ 1 · contradiction and conclusion ½.
3. **$a + b\sqrt{p}$ is irrational:** assume rational ½ · make $\sqrt{p}$ the subject 1 · say the right side is rational, so contradiction ½ (3-mark versions give 1 more for the reasoning).
4. **HCF and LCM:** prime factorisation 1 · HCF ½ · LCM ½; HCF × LCM = product for the missing one.
5. **Word problems:** deciding HCF or LCM (with the reason) · working · the final answer in the units or time asked.
6. Lines with no number are working that earns no separate mark but must be shown.

A

PART A · SOLUTIONS

# Guided Practice – Solutions

Worked solutions with step marks for every question in Part A.

## Prime factorisation, HCF and LCM

**A1.** Complete the division ladder to write 360 as a product of primes. [2]

$360 \div 2 = 180, 180 \div 2 = 90, 90 \div 2 = 45, 45 \div 3 = 15, 15 \div 3 = 5, 5 \div 5 = 1$ — 1

$360 = 2^3 \times 3^2 \times 5$ — 1

**A2.** Complete the steps to find the HCF and LCM of 84 and 120. [3]

$84 = 2^2 \times 3 \times 7, 120 = 2^3 \times 3 \times 5$ — 1

$\text{HCF} = 2^2 \times 3 = 12$ — 1

$\text{LCM} = 2^3 \times 3 \times 5 \times 7 = 840$ (check: $12 \times 840 = 10080 = 84 \times 120$ ✓) — 1

## Irrationality proofs

**A3.** Complete the proof that $\sqrt{2}$ is irrational. [3]

Assume $\sqrt{2} = \frac{a}{b}$, where $a, b$ are coprime integers and $b \neq 0$ — ½

$2b^2 = a^2$, so 2 divides $a^2$, hence 2 divides $a$ — 1

$a = 2c$: $2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$, so 2 divides $b$ — 1

2 is a common factor of $a$ and $b$: contradiction. So $\sqrt{2}$ is irrational — ½

**A4.** Given that $\sqrt{3}$ is irrational, complete the proof that $5 + 2\sqrt{3}$ is irrational. [2]

Assume $5 + 2\sqrt{3} = \frac{a}{b}$ ($a$, $b$ coprime integers, $b \neq 0$): $\sqrt{3} = \frac{a - 5b}{2b}$ — 1

$\frac{a - 5b}{2b}$ is rational, so $\sqrt{3}$ would be rational: contradiction. So $5 + 2\sqrt{3}$ is irrational — 1

## Using prime factors to reason

**A5.** Complete the reasoning: can $6^n$ end with the digit 0 for any natural number $n$? [2]

$6^n = 2^n \times 3^n$. To end with 0, a number must be divisible by $10 = 2 \times 5$ — 1

5 is not a prime factor of $6^n$, so $6^n$ can never end with 0 — 1

**A6.** Write HCF or LCM for each situation. **(i)** Three bells ring every 10, 15 and 20 minutes. When do they ring together again? **(ii)** The largest square tile that exactly covers a 4 m by 6 m floor. **(iii)** The smallest number of sweets that can be shared equally among 8 or among 12 children. **(iv)** The greatest number of identical gift packs from 24 pens and 36 pencils. [2]

(i) LCM — ½

(ii) HCF — ½

(iii) LCM — ½

(iv) HCF — ½

# B PART B · SOLUTIONS
# NCERT Practice – Solutions

Worked solutions with step marks for every question in Part B.

## Prime factorisation (Fundamental Theorem of Arithmetic)

**B1.** Express each number as a product of its prime factors: **(i)** 180 **(ii)** 252 **(iii)** 4725 **(iv)** 6435 **(v)** 4199 [5]

(i) $180 = 2^2 \times 3^2 \times 5$ — 1

(ii) $252 = 2^2 \times 3^2 \times 7$ — 1

(iii) $4725 = 3^3 \times 5^2 \times 7$ — 1

(iv) $6435 = 3^2 \times 5 \times 11 \times 13$ — 1

(v) $4199 = 13 \times 17 \times 19$ — 1

| Q | Answer | Working / reason |
|---|---|---|
| B2 | **(c)** 4 | $144 = 2^4 \times 3^2$, so the exponent of 2 is 4. |

**B3.** Check whether $8^n$ can end with the digit 0 for any natural number $n$. [2]

$8^n = (2^3)^n = 2^{3n}$ — ½

A number ending in 0 is divisible by $10 = 2 \times 5$, so 5 must be a prime factor — ½

By the Fundamental Theorem of Arithmetic the only prime factor of $8^n$ is 2; 5 is not a factor — ½

So $8^n$ can never end with 0 — ½

**B4.** Explain why $5 \times 11 \times 17 + 17$ and $6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6$ are composite numbers. [2]

$5 \times 11 \times 17 + 17 = 17(5 \times 11 + 1) = 17 \times 56$: it has factors other than 1 and itself, so it is composite — 1

$6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6 = 6(120 + 1) = 6 \times 121$: composite for the same reason — 1

## HCF and LCM by prime factorisation

**B5.** Find the HCF and LCM of 12 and 42 by the prime factorisation method. [2]

$12 = 2^2 \times 3$, $42 = 2 \times 3 \times 7$ — 1

$\text{HCF} = 2 \times 3 = 6$ — ½

$\text{LCM} = 2^2 \times 3 \times 7 = 84$ — ½

**B6.** Find the LCM and HCF of each pair and verify that LCM × HCF = product of the two numbers: **(i)** 38 and 57 **(ii)** 390 and 76 **(iii)** 288 and 120 [3]

(i) $38 = 2 \times 19$, $57 = 3 \times 19$: HCF $= 19$, LCM $= 114$; $19 \times 114 = 2166 = 38 \times 57$ ✓ — 1

(ii) $390 = 2 \times 3 \times 5 \times 13$, $76 = 2^2 \times 19$: HCF $= 2$, LCM $= 14820$; $2 \times 14820 = 29640 = 390 \times 76$ ✓ — 1

(iii) $288 = 2^5 \times 3^2$, $120 = 2^3 \times 3 \times 5$: HCF $= 24$, LCM $= 1440$; $24 \times 1440 = 34560 = 288 \times 120$ ✓ — 1

**B7.** Find the LCM and HCF of the following integers by the prime factorisation method: **(i)** 18, 24 and 30 **(ii)** 13, 19 and 31 **(iii)** 4, 27 and 35 [3]

(i) $18 = 2 \times 3^2$, $24 = 2^3 \times 3$, $30 = 2 \times 3 \times 5$: HCF $= 6$, LCM $= 2^3 \times 3^2 \times 5 = 360$ — 1

(ii) 13, 19, 31 are primes: HCF $= 1$, LCM $= 13 \times 19 \times 31 = 7657$ — 1

(iii) $4 = 2^2$, $27 = 3^3$, $35 = 5 \times 7$: HCF $= 1$, LCM $= 4 \times 27 \times 35 = 3780$ — 1

**B8.** Given that HCF(252, 594) = 18, find LCM(252, 594). [2]

$$\text{LCM} = \frac{252 \times 594}{\text{HCF}} = \frac{252 \times 594}{18}$$ — 1

$= 14 \times 594 = 8316$ — 1

**B9.** Two cyclists start together from the same point of a circular track and ride in the same direction. One completes a round in 16 minutes and the other in 20 minutes. After how many minutes will they meet again at the starting point? [2]

They meet when both have done whole rounds: the LCM of 16 and 20 — ½

$16 = 2^4$, $20 = 2^2 \times 5$ — ½

$\text{LCM} = 2^4 \times 5 = 80$: they meet after 80 minutes — 1

## Irrational numbers

**B10.** Prove that $\sqrt{5}$ is irrational. **[3]**

Assume, to the contrary, that $\sqrt{5}$ is rational: $\sqrt{5} = \frac{a}{b}$, where $a$, $b$ are coprime integers and $b \neq 0$ — ½

Squaring, $5b^2 = a^2$. So 5 divides $a^2$, and hence 5 divides $a$ — 1

Let $a = 5c$. Then $5b^2 = 25c^2$, i.e. $b^2 = 5c^2$. So 5 divides $b^2$, and hence 5 divides $b$ — 1

5 is a common factor of $a$ and $b$, which contradicts that they are coprime. So $\sqrt{5}$ is irrational — ½

**B11.** Given that $\sqrt{3}$ is irrational, prove that $2 + 5\sqrt{3}$ is irrational. **[3]**

Assume $2 + 5\sqrt{3}$ is rational: $2 + 5\sqrt{3} = \frac{a}{b}$, where $a$, $b$ are coprime integers, $b \neq 0$ — ½

$5\sqrt{3} = \frac{a}{b} - 2 = \frac{a - 2b}{b}$, so $\sqrt{3} = \frac{a - 2b}{5b}$ — 1

$a$, $b$ are integers, so $\frac{a - 2b}{5b}$ is rational, and then $\sqrt{3}$ would be rational — 1

This contradicts the fact that $\sqrt{3}$ is irrational. Hence $2 + 5\sqrt{3}$ is irrational — ½

**B12.** Prove that the following are irrational: **(i)** $\frac{1}{\sqrt{3}}$ **(ii)** $4\sqrt{2}$ **(iii)** $7 - \sqrt{5}$ **[3]**

(i) If $\frac{1}{\sqrt{3}} = \frac{a}{b}$ ($a, b$ integers, $a, b \neq 0$), then $\sqrt{3} = \frac{b}{a}$ is rational: contradiction. So $\frac{1}{\sqrt{3}}$ is irrational — 1

(ii) If $4\sqrt{2} = \frac{a}{b}$, then $\sqrt{2} = \frac{a}{4b}$ is rational: contradiction. So $4\sqrt{2}$ is irrational — 1

(iii) If $7 - \sqrt{5} = \frac{a}{b}$, then $\sqrt{5} = \frac{7b - a}{b}$ is rational: contradiction. So $7 - \sqrt{5}$ is irrational — 1

| Q | Answer | Working / reason |
|---|---|---|
| B13 | **(b)** 3 divides a | If a prime $p$ divides $a^2$, then $p$ divides $a$. Here $p = 3$. ($a = 3$ and $a = 6$ show that the other options need not be true.) |

# C PART C · SOLUTIONS
# Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

## Irrationality proofs

**C1.** Prove that $\sqrt{3}$ is irrational. **[3]**

Assume, to the contrary, that $\sqrt{3} = \frac{a}{b}$, where $a$, $b$ are coprime integers and $b \neq 0$ — ½

Squaring, $3b^2 = a^2$. So 3 divides $a^2$, and hence 3 divides $a$ — 1

Let $a = 3c$. Then $3b^2 = 9c^2$, i.e. $b^2 = 3c^2$. So 3 divides $b^2$, and hence 3 divides $b$ — 1

3 is a common factor of $a$ and $b$: this contradicts that they are coprime. So $\sqrt{3}$ is irrational — ½

**C2.** Given that $\sqrt{5}$ is irrational, prove that $4 - 3\sqrt{5}$ is irrational. [2]

Assume $4 - 3\sqrt{5} = \frac{a}{b}$, where $a$, $b$ are coprime integers, $b \neq 0$ ½

$3\sqrt{5} = 4 - \frac{a}{b} = \frac{4b - a}{b}$, so $\sqrt{5} = \frac{4b - a}{3b}$ 1

The right side is rational, so $\sqrt{5}$ would be rational: contradiction. Hence $4 - 3\sqrt{5}$ is irrational ½

**C3.** Given that $\sqrt{5}$ is irrational, prove that $\frac{3}{\sqrt{5}}$ is irrational. [2]

Assume $\frac{3}{\sqrt{5}} = \frac{a}{b}$, where $a$, $b$ are coprime integers, $a \neq 0$, $b \neq 0$ ½

Then $\sqrt{5} = \frac{3b}{a}$ 1

$\frac{3b}{a}$ is rational, so $\sqrt{5}$ would be rational: contradiction. Hence $\frac{3}{\sqrt{5}}$ is irrational ½

**C4.** Given that $\sqrt{2}$ is irrational, prove that $3\sqrt{2} - 1$ is irrational. [2]

Assume $3\sqrt{2} - 1 = \frac{a}{b}$, where $a$, $b$ are coprime integers, $b \neq 0$ ½

$3\sqrt{2} = \frac{a + b}{b}$, so $\sqrt{2} = \frac{a + b}{3b}$ 1

The right side is rational, so $\sqrt{2}$ would be rational: contradiction. Hence $3\sqrt{2} - 1$ is irrational ½

## Finding HCF and LCM

| Q | Answer | Working / reason |
|---|---|---|
| C5 | **(b)** 252 | $756 = 2^2 \times 3^3 \times 7$, $504 = 2^3 \times 3^2 \times 7 \rightarrow$ HCF $= 2^2 \times 3^2 \times 7 = 252$ |
| C6 | **(d)** 216 | Greatest powers: $2^3 \times 3^3 = 216$ |
| C7 | **(b)** $30a^4b^3$ | $15 = 3 \cdot 5$, $10 = 2 \cdot 5 \rightarrow 2 \cdot 3 \cdot 5 = 30$; greatest powers $a^4$, $b^3 \rightarrow 30a^4b^3$ |
| C8 | **(c)** 150 | LCM $= \frac{1800}{12} = 150$ |
| C9 | **(c)** pq | Coprime $\rightarrow$ HCF $= 1$, so LCM $= \frac{pq}{1} = pq$. |
| C10 | **(c)** 100 | LCM $= \frac{1080 \times 1200}{120} = 10800 = 108 \times 100$ |
| C11 | **(d)** | $380 \div 16 = 23.75$ is not whole, so A is false. R is true. |

**C12.** The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other. [2]

Other number $= \frac{23 \times 1449}{161}$ 1

$= \frac{33327}{161} = 207$ 1

**C13.** Find the HCF of 120 and 144 by prime factorisation. Hence find their LCM. [2]

$120 = 2^3 \times 3 \times 5$, $144 = 2^4 \times 3^2$ ½

HCF $= 2^3 \times 3 = 24$ ½

LCM $= \frac{120 \times 144}{24} = 720$ 1

**C14.** Find the HCF and LCM of 72, 108 and 180 by prime factorisation. Is HCF × LCM equal to the product of the three numbers? Justify. [3]

$72 = 2^3 \times 3^2$, $108 = 2^2 \times 3^3$, $180 = 2^2 \times 3^2 \times 5$ — 1

HCF $= 2^2 \times 3^2 = 36$ — ½

LCM $= 2^3 \times 3^3 \times 5 = 1080$ — ½

HCF × LCM $= 38880$ but $72 \times 108 \times 180 = 1399680$. Not equal: the rule holds for two numbers only — 1

## HCF and LCM word problems

**C15.** Three school bells ring at intervals of 12, 18 and 30 minutes. If they ring together at 8:00 a.m., at what time will they next ring together? [2]

$12 = 2^2 \times 3$, $18 = 2 \times 3^2$, $30 = 2 \times 3 \times 5$: LCM $= 2^2 \times 3^2 \times 5 = 180$ min — 1

180 min = 3 hours, so they ring together again at 11:00 a.m. — 1

**C16.** A school workshop has 45 Science, 75 Maths and 105 English teachers. Each room must seat the same number of teachers, all of one subject. Find the minimum number of rooms needed. [3]

Fewest rooms → most teachers per room, a number dividing 45, 75 and 105: the HCF — ½

$45 = 3^2 \times 5$, $75 = 3 \times 5^2$, $105 = 3 \times 5 \times 7$ — 1

HCF $= 3 \times 5 = 15$ teachers per room — ½

Rooms $= \dfrac{45 + 75 + 105}{15} = \dfrac{225}{15} = 15$ — 1

**C17.** Find the greatest length of a measuring rod that can measure 3 m 85 cm and 5 m 95 cm an exact number of times. [2]

385 cm and 595 cm: $385 = 5 \times 7 \times 11$, $595 = 5 \times 7 \times 17$ — 1

HCF $= 5 \times 7 = 35$: the rod is 35 cm long — 1

**C18.** Find the greatest number which divides 245 and 1029, leaving a remainder 5 in each case. [2]

The number divides $245 - 5 = 240$ and $1029 - 5 = 1024$ exactly — ½

$240 = 2^4 \times 3 \times 5$, $1024 = 2^{10}$ — ½

HCF $= 2^4 = 16$ — 1

**C19.** Find the smallest number which, when increased by 13, is exactly divisible by both 280 and 504. [2]

$280 = 2^3 \times 5 \times 7$, $504 = 2^3 \times 3^2 \times 7$: LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$ — 1

Required number $= 2520 - 13 = 2507$ — 1

## Prime and composite numbers · factor trees

| Q | Answer | Working / reason |
|---|---|---|
| C20 | **(c)** 2 is the only even prime number | 1 is neither prime nor composite; 4 is the smallest composite; 9 is odd but not prime. |

| | | |
|---|---|---|
| **C21** | **(b)** a composite number | $7(5 \times 11 + 1) = 7 \times 56 = 392$: it has factors other than 1 and itself. |

**C22.** Show that $2 \times 3 \times 5 \times 7 \times 11 + 7 \times 11$ is a composite number. **[2]**

$= 7 \times 11\,(2 \times 3 \times 5 + 1) = 7 \times 11 \times 31$ — 1

It has factors 7, 11, 31 other than 1 and itself, so it is composite — 1

| Q | Answer | Working / reason |
|---|---|---|
| **C23** | **(a)** | $13(77 + 1) = 13 \times 78$, so A is true, and R is exactly the reason. |

**C24.** In a factor tree, $N$ splits as $2 \times a$, $a$ splits as $3 \times b$, and $b$ splits as $7 \times 13$. Find $b$, $a$ and $N$, and write $N$ as a product of primes. **[2]**

$b = 7 \times 13 = 91$; $a = 3 \times 91 = 273$ — 1

$N = 2 \times 273 = 546 = 2 \times 3 \times 7 \times 13$ — 1

## Last digit · numbers ending in 0

| Q | Answer | Working / reason |
|---|---|---|
| **C25** | **(a)** 1 | $5^n$ always ends in 5 and $6^n$ always ends in 6; $5 + 6 = 11 \rightarrow$ last digit 1. |
| **C26** | **(d)** $20^n$ | Only $20^n = 2^{2n} \times 5^n$ has both 2 and 5 as factors. |

**C27.** Check whether $35^n$ can end with the digit 0 for any natural number $n$. Justify your answer. **[2]**

$35^n = 5^n \times 7^n$ — ½

To end with 0 a number must have both 2 and 5 as prime factors — ½

2 is not a prime factor of $35^n$ (prime factorisation is unique), so $35^n$ never ends with 0 — 1

## Rational or irrational? Number facts

| Q | Answer | Working / reason |
|---|---|---|
| **C28** | **(b)** 1.5 | $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$: only 1.5 lies between them and is rational ($\sqrt{2.5}$ is irrational). |
| **C29** | **(d)** $(1 + \sqrt{3})^2$ | $(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$. The others are 1, 5 and 6. |
| **C30** | **(a)** | Both are true and R is the reason for A. |
| **C31** | **(d)** an irrational number | rational – irrational = irrational. |

PART D · SOLUTIONS

# D Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

## D1. Case Study · Sports Meet [4]

**(i)** After how many minutes will Aarav and Bhavya first meet again at the starting point? **[1]**

$\text{LCM}(6, 9) = 2 \times 3^2 = 18$ minutes — 1

**(ii)** Find the HCF of 6, 9 and 15. **[1]**

$6 = 2 \times 3$, $9 = 3^2$, $15 = 3 \times 5$: $\text{HCF} = 3$ — 1

**(iii)** At what time will all three be at the starting point together again? **[2]**

$\text{LCM}(6, 9, 15) = 2 \times 3^2 \times 5 = 90$ minutes — 1

90 min = 1 h 30 min after 7:00 → 8:30 a.m. — 1

OR

**(iii)** How many rounds will Charu have completed when all three first meet again at the starting point? **[2]**

They meet after LCM = 90 minutes — 1

$\text{Charu's rounds} = \dfrac{90}{15} = 6$ — 1

## D2. Case Study · Donation Drive [4]

**(i)** What is the greatest number of identical kits they can make? **[1]**

$336 = 2^4 \cdot 3 \cdot 7$, $240 = 2^4 \cdot 3 \cdot 5$, $96 = 2^5 \cdot 3$: $\text{HCF} = 2^4 \times 3 = 48$ kits — 1

**(ii)** How many notebooks will there be in each kit? **[1]**

$\dfrac{336}{48} = 7$ notebooks — 1

**(iii)** Write 336 and 240 as products of primes and find their LCM. **[2]**

$336 = 2^4 \times 3 \times 7$, $240 = 2^4 \times 3 \times 5$ — 1

$\text{LCM} = 2^4 \times 3 \times 5 \times 7 = 1680$ — 1

OR

**(iii)** If they had collected 252 pencils instead of 240, what would be the greatest number of identical kits? **[2]**

$252 = 2^2 \times 3^2 \times 7$ — 1

$\text{HCF}(336, 252, 96) = 2^2 \times 3 = 12$ kits — 1

## D3. Case Study · Factor Tree [4]

**(i)** Find the values of $c$ and $b$. **[1]**

$c = 7 \times 11 = 77$; $b = 5 \times 77 = 385$ — 1

**(ii)** Find $N$ and write it as a product of primes. **[1]**

$a = 3 \times 385 = 1155$; $N = 2 \times 1155 = 2310 = 2 \times 3 \times 5 \times 7 \times 11$ — 1

**(iii)** Find the HCF and LCM of $N$ and 840 by prime factorisation. **[2]**

$840 = 2^3 \times 3 \times 5 \times 7$, $N = 2 \times 3 \times 5 \times 7 \times 11$ ½

HCF $= 2 \times 3 \times 5 \times 7 = 210$ ½

LCM $= 2^3 \times 3 \times 5 \times 7 \times 11 = 9240$ 1

OR

**(iii)** Find the smallest number divisible by each of 2, 3, 4, 5, 7 and 11. Is it equal to $N$? Give a reason. **[2]**

LCM $= 2^2 \times 3 \times 5 \times 7 \times 11 = 4620$ 1

No: $4620 = 2 \times 2310$. $N$ has only one factor 2, so it is not divisible by 4 1