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CLASS X · MATHEMATICS STANDARD (041) · CBSE BOARD EXAMINATION 2027

CHAPTER 6

Triangles

Answer Booklet · Full worked solutions with CBSE-style step marks

Every question of the Standard booklet | Step marks shown line by line | MCQ keys with working

INSIDE THIS BOOKLET

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HOW TO USE THE ANSWERS

1. Solve the question in your notebook **first**. Open this booklet only to check.
2. The orange number beside each line is the **step mark** an examiner gives for that line, as in the CBSE marking scheme.
3. Give yourself marks line by line. A right final answer without the working can lose step marks.
4. Check each similarity statement by matching the equal angles letter by letter before you write the ratio.
5. Copy every wrong answer to the My Mistakes page of your booklet.

READ THIS FIRST

Marking Guide

Full worked solutions to every question in the chapter booklet, set out the way a CBSE examiner marks them.

HOW MARKS ARE GIVEN IN TRIANGLES ANSWERS

1. **1-mark MCQ / assertion–reason:** only the correct option scores. The working here is for your understanding.
2. **Similarity in a figure (2 marks):** two pairs of equal angles with reasons 1 · criterion and the required ratio 1.
3. **BPT numerical (2 marks):** correct proportion 1 · value 1.
4. **Prove the BPT (3 marks):** given, to prove, construction and figure ½ + ½ · area ratios 1 · equal areas and conclusion 1.
5. **5-mark question:** usually the BPT proof (3) plus an application (2).
6. Lines with no number are working that earns no separate mark but must be shown.

PART A · SOLUTIONS

A NCERT Practice – Solutions

Worked solutions with step marks for every question in Part A.

Similar figures

Q	Answer	Working / reason
A1	**(c)** two equilateral triangles	All angles of an equilateral triangle are 60°, and all its sides are equal, so any two are similar. The others can have different shapes.

A2. Give a counter-example for each statement: **(i)** Two quadrilaterals with equal corresponding angles need not be similar. **(ii)** Two quadrilaterals with proportional sides need not be similar. [2]

(i) A square of side 2 and a rectangle 2 × 4: all angles 90°, but $\frac{2}{2} \neq \frac{2}{4}$ — 1

(ii) A square of side 3 and a rhombus of side 3 with angles 60° and 120°: sides in ratio 1 : 1, but angles differ — 1

Basic Proportionality Theorem

A3. In $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC. Find: **(i)** EC, if $AD = 2$ cm, $DB = 3$ cm, $AE = 2.4$ cm **(ii)** AE, if $AD = 4$ cm, $AB = 10$ cm, $AC = 7.5$ cm [2]

(i) $\frac{AD}{DB} = \frac{AE}{EC} \rightarrow \frac{2}{3} = \frac{2.4}{EC} \rightarrow EC = 3.6$ cm — 1

(ii) $\frac{AD}{AB} = \frac{AE}{AC} \rightarrow \frac{4}{10} = \frac{AE}{7.5} \rightarrow AE = 3$ cm — 1

A4. E and F are points on sides PQ and PR of $\triangle PQR$. Is $EF \parallel QR$? **(i)** $PE = 4$, $EQ = 5$, $PF = 3.2$, $FR = 4$ (cm) **(ii)** $PE = 3$, $EQ = 4.5$, $PF = 2.4$, $FR = 3$ (cm) [2]

(i) $\frac{PE}{EQ} = \frac{4}{5} = 0.8$, $\frac{PF}{FR} = \frac{3.2}{4} = 0.8$: equal, so **yes**, $EF \parallel QR$ — 1

(ii) $\frac{PE}{EQ} = \frac{3}{4.5} = \frac{2}{3}$, $\frac{PF}{FR} = \frac{2.4}{3} = 0.8$: not equal, so **no** — 1

A5. In the figure, $LM \parallel CB$ and $LN \parallel CD$. Prove that $\frac{AM}{AB} = \frac{AN}{AD}$. **[3]**

In $\triangle ABC$, $LM \parallel CB$, so by the BPT $\frac{AM}{AB} = \frac{AL}{AC}$ — 1

In $\triangle ADC$, $LN \parallel CD$, so $\frac{AN}{AD} = \frac{AL}{AC}$ — 1

Hence $\frac{AM}{AB} = \frac{AN}{AD}$ — 1

A6. Using the BPT, prove that a line drawn through the midpoint of one side of a triangle, parallel to another side, bisects the third side. **[2]**

D is the midpoint of AB and $DE \parallel BC$. By the BPT $\frac{AE}{EC} = \frac{AD}{DB} = 1$ — 1

So $AE = EC$: E is the midpoint of AC — 1

A7. The diagonals of quadrilateral $ABCD$ meet at O and $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium. **[3]**

Draw $OE \parallel AB$ meeting AD at E. In $\triangle ABD$: $\frac{AE}{ED} = \frac{BO}{OD}$ — 1

Given $\frac{AO}{BO} = \frac{CO}{DO}$, i.e. $\frac{AO}{OC} = \frac{BO}{OD}$. So $\frac{AE}{ED} = \frac{AO}{OC}$ — 1

By the converse of the BPT in $\triangle ADC$, $EO \parallel DC$. As $EO \parallel AB$ too, $AB \parallel DC$: a trapezium — 1

Similarity criteria

A8. State which pairs of triangles are similar, and write the criterion: **(i)** $AB = 2$, $BC = 2.5$, $CA = 3$; $PQ = 4$, $QR = 5$, $RP = 6$ **(ii)** $\angle L = 50^\circ$, $\angle M = 70^\circ$; $\angle X = 50^\circ$, $\angle Z = 60^\circ$ **(iii)** $DE = 3$, $EF = 4$, $\angle E = 40^\circ$; $ST = 6$, $TU = 10$, $\angle T = 40^\circ$ **[3]**

(i) $\frac{2}{4} = \frac{2.5}{5} = \frac{3}{6} = \frac{1}{2}$: $\triangle ABC \sim \triangle PQR$ (SSS) — 1

(ii) $\angle Y = 180^\circ - 50^\circ - 60^\circ = 70^\circ$; $\angle L = \angle X$, $\angle M = \angle Y$: $\triangle LMN \sim \triangle XYZ$ (AA) — 1

(iii) $\angle E = \angle T$ but $\frac{3}{6} \neq \frac{4}{10}$: **not** similar — 1

A9. In the figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 115^\circ$ and $\angle CDO = 75^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$. **[3]**

$\angle DOC = 180^\circ - 115^\circ = 65^\circ$ (linear pair) — 1

$\angle DCO = 180^\circ - 65^\circ - 75^\circ = 40^\circ$ — 1

$\angle OAB = \angle OCD = 40^\circ$ (similar triangles) — 1

A10. In $\triangle PQR$, $\angle Q = 90^\circ$ and $QS \perp PR$. Prove that $\triangle PSQ \sim \triangle PQR$ and $PQ^2 = PS \times PR$. **[3]**

In $\triangle PSQ$ and $\triangle PQR$: $\angle P$ is common and $\angle PSQ = \angle PQR = 90^\circ$ — 1

$\therefore \triangle PSQ \sim \triangle PQR$ (AA) — 1

$\frac{PS}{PQ} = \frac{PQ}{PR}$, so $PQ^2 = PS \times PR$ — 1

A11. A girl 1.5 m tall walks away from the foot of a lamp post at 1 m/s. The lamp is 4.5 m above the ground. How long is her shadow after 6 seconds? [3]

After 6 s she is 6 m from the post. Let the shadow be s m. The triangles are similar (AA) 1

$\frac{1.5}{4.5} = \frac{s}{s+6} \rightarrow 3s = s + 6$ 1

$s = 3$ m 1

A12. A stick 2 m long, held upright, casts a shadow 1.6 m long. At the same time a building casts a shadow 24 m long. Find the height of the building. [2]

The triangles are similar (AA): $\frac{h}{2} = \frac{24}{1.6}$ 1

$h = 30$ m 1

A13. AD and PS are medians of $\triangle ABC$ and $\triangle PQR$, and $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$. [3]

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QS} = \frac{BD}{QS}$ and $\angle B = \angle Q$ 1

$\therefore \triangle ABD \sim \triangle PQS$ (SAS) 1

Hence $\frac{AB}{PQ} = \frac{AD}{PS}$ 1

Pythagoras (application)

A14. A ladder 13 m long reaches a window 12 m above the ground. How far is the foot of the ladder from the wall? [2]

$d^2 = 13^2 - 12^2 = 169 - 144 = 25$ 1

$d = 5$ m 1

B

PART B · SOLUTIONS

Exemplar Practice – Solutions

Worked solutions with step marks for every question in Part B.

Multiple choice

Q	Answer	Working / reason
B1	**(d)** PQ · YX = PR · ZX	$P \leftrightarrow Z, Q \leftrightarrow Y, R \leftrightarrow X$: $\frac{PQ}{ZY} = \frac{QR}{YX} = \frac{PR}{ZX}$. (d) would need $\frac{PQ}{ZX} = \frac{PR}{YX}$, which does not follow.
B2	**(b)** 1 : 3	$\triangle ADE \sim \triangle ABC$, so $\frac{DE}{BC} = \frac{AD}{AB} = \frac{2}{6}$.
B3	**(a)** 28 cm	$21 \times \frac{8}{6} = 28$ cm.
B4	**(b)** BD · CD = AD^2	$\triangle BDA \sim \triangle ADC$ gives $\frac{BD}{AD} = \frac{AD}{CD}$.
B5	**(c)** $\frac{KL}{XY} = \frac{LM}{ZX}$	$K \leftrightarrow Y, M \leftrightarrow X, L \leftrightarrow Z$: KL goes with YZ, not XY.
B6	**(c)** DE = 9 cm, ∠F = 80°	$F \leftrightarrow B$: $\angle F = \angle B = 80^\circ$. Ratio $\frac{DF}{AB} = 1.5$, so $DE = 1.5 \times AC = 9$ cm.
B7	**(b)** 1 : 4	$\frac{2}{6} = \frac{3}{9}$, so $ST \parallel QR$ and $\frac{ST}{QR} = \frac{PS}{PQ} = \frac{2}{8}$.

Short answer with reasons

B8. Is a triangle with sides 10 cm, 25 cm and 26 cm a right triangle? Justify. [2]

$10^2 + 25^2 = 100 + 625 = 725$ and $26^2 = 676$ — 1

They are not equal, so it is **not** a right triangle — 1

B9. Two sides and the perimeter of one triangle are three times the corresponding sides and perimeter of another. Are the triangles similar? Justify. [2]

Sides $3a$, $3b$ and perimeter $3(a+b+c)$, so the third side is $3c$ — 1

All three sides are in the ratio 3 : 1: the triangles are similar (SSS) — 1

B10. $\triangle ABC \sim \triangle RQP$, $\angle A = 80^\circ$ and $\angle B = 60^\circ$. Find $\angle Q$ and $\angle P$. [2]

$B \leftrightarrow Q$: $\angle Q = 60^\circ$ — 1

$C \leftrightarrow P$: $\angle P = \angle C = 180^\circ - 140^\circ = 40^\circ$ — 1

B11. In the figure, $\angle ADE = \angle ACB$. Show that $\triangle ADE \sim \triangle ACB$. If $AD = 3$ cm, $AE = 4$ cm and $AC = 9$ cm, find AB. [3]

$\angle A$ is common and $\angle ADE = \angle ACB$: $\triangle ADE \sim \triangle ACB$ (AA) — 1

$\frac{AD}{AC} = \frac{AE}{AB} \rightarrow \frac{3}{9} = \frac{4}{AB}$ — 1

$AB = 12$ cm — 1

B12. $ABCD$ is a trapezium with $AB \parallel DC$ and $AB = 2\,CD$. Its diagonals meet at O and $AO = 6$ cm. Find OC and AC. **[2]**

$\triangle AOB \sim \triangle COD$, so $\dfrac{AO}{OC} = \dfrac{AB}{CD} = 2$ — 1

$OC = 3$ cm, $AC = 9$ cm — 1

Long answer

B13. Two vertical poles 6 m and 3 m high stand on level ground. The top of each pole is joined to the foot of the other. How high above the ground do the two lines cross? **[4]**

Poles $AP = x = 6$, $CR = y = 3$; the lines cross at Q, $QB \perp AC$, $QB = z$. In $\triangle CAP$, $QB \parallel PA$: $\dfrac{z}{x} = \dfrac{CB}{CA}$ — 1

In $\triangle ACR$, $QB \parallel RC$: $\dfrac{z}{y} = \dfrac{AB}{AC}$ — 1

Add: $\dfrac{z}{x} + \dfrac{z}{y} = \dfrac{CB + AB}{AC} = 1$, so $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$ — 1

$\dfrac{1}{z} = \dfrac{1}{6} + \dfrac{1}{3} = \dfrac{1}{2} \rightarrow z = 2$ m — 1

B14. D is a point on side BC of $\triangle ABC$ with $\angle ADC = \angle BAC$. Show that $CA^2 = CB \times CD$. If $CB = 9$ cm and $CD = 4$ cm, find CA. **[4]**

In $\triangle ADC$ and $\triangle BAC$: $\angle C$ is common and $\angle ADC = \angle BAC$ — 1

$\therefore \triangle ADC \sim \triangle BAC$ (AA), so $\dfrac{CA}{CB} = \dfrac{CD}{CA}$ — 1

$CA^2 = CB \times CD$ — 1

$CA^2 = 36 \rightarrow CA = 6$ cm — 1

B15. $ABCD$ is a parallelogram. A line through B meets DC at P and AD produced at Q. Prove that $\triangle BCP \sim \triangle QDP$ and hence $\dfrac{CP}{DP} = \dfrac{BC}{QD}$. **[3]**

$\angle BPC = \angle QPD$ (vertically opposite) — 1

$\angle PBC = \angle PQD$ (alternate angles, $BC \parallel AQ$); so $\triangle BCP \sim \triangle QDP$ (AA) — 1

Hence $\dfrac{CP}{DP} = \dfrac{BC}{QD}$ — 1

C

PART C · SOLUTIONS

Board Pattern Practice – Solutions

Worked solutions with step marks for every question in Part C.

Sides and perimeters of similar triangles

Q	Answer	Working / reason
C1	**(b)** 12 cm	$PQ = 9 \times \frac{48}{36} = 12$ cm.
C2	**(c)** 15 cm	$\frac{YZ}{QR} = \frac{XY}{PQ} = 3$, so $YZ = 15$ cm.
C3	**(a)** $\triangle$ABC ~ $\triangle$EDF	SAS with $A \leftrightarrow E$, $B \leftrightarrow D$, $C \leftrightarrow F$.
C4	**(d)** RHS	RHS is a congruence rule.
C5	**(b)** similar but not congruent	AA gives similarity; the sides are not equal, so they are not congruent.
C6	**(d)** 12 cm	$8 \times \frac{9}{6} = 12$ cm.
C7	**(a)**	$\frac{4}{6} = \frac{2}{3}$; R explains A.

BPT: lengths and parallel lines

Q	Answer	Working / reason
C8	**(d)** 6 cm	$\frac{3}{4} = \frac{4.5}{EC} \rightarrow EC = 6$ cm.
C9	**(b)** 4.5 cm	$\frac{AE}{AC} = \frac{3}{8}$, so $AE = 4.5$ cm.
C10	**(c)** 2	$\frac{4}{6} = \frac{3x}{9} \rightarrow x = 2$.
C11	**(a)**	$\frac{AD}{DB} = \frac{AE}{EC} = 1$; by the converse of the BPT, $DE \parallel BC$. R explains A.

C12. In $\triangle ABC$, $DE \parallel BC$ with $AD = x + 2$, $DB = x - 1$, $AE = x + 5$ and $EC = x + 1$ (cm). Find x. **[2]**

$\frac{x+2}{x-1} = \frac{x+5}{x+1} \rightarrow x^2 + 3x + 2 = x^2 + 4x - 5$ — 1

$x = 7$ (check: $\frac{9}{6} = \frac{12}{8}$ ✓) — 1

C13. D and E are points on AB and AC of $\triangle ABC$ with $AD = 1.5$, $DB = 3$, $AE = 1$ and $EC = 2$ (cm). Show that $DE \parallel BC$. **[2]**

$\frac{AD}{DB} = \frac{1.5}{3} = \frac{1}{2}$ and $\frac{AE}{EC} = \frac{1}{2}$ — 1

The ratios are equal, so $DE \parallel BC$ (converse of the BPT) — 1

C14. In the figure, $LM \parallel AB$. If $AL = x - 3$, $AC = 2x$, $BM = x - 2$ and $BC = 2x + 3$, find x. **[2]**

$\frac{CL}{CA} = \frac{CM}{CB}$: $\frac{x+3}{2x} = \frac{x+5}{2x+3}$ — 1

$2x^2 + 9x + 9 = 2x^2 + 10x \rightarrow x = 9$ — 1

Show two triangles similar

C15. In the figure, $AB \parallel CD$ and AD, BC meet at O. Show that $\triangle OAB \sim \triangle ODC$. If $OA = 3$ cm, $OD = 5$ cm and $AB = 4.2$ cm, find CD. [2]

$\angle AOB = \angle DOC$ (vertically opposite), $\angle OAB = \angle ODC$ (alternate): $\triangle OAB \sim \triangle ODC$ (AA) 1

$\frac{AB}{DC} = \frac{OA}{OD} \rightarrow \frac{4.2}{CD} = \frac{3}{5} \rightarrow CD = 7$ cm 1

C16. In the figure, $QA \perp AB$ and $PB \perp AB$. If $AO = 8$ cm, $BO = 6$ cm and $PB = 4.5$ cm, find AQ. [2]

$\angle A = \angle B = 90^\circ$, $\angle AOQ = \angle BOP$: $\triangle AOQ \sim \triangle BOP$ (AA) 1

$\frac{AQ}{BP} = \frac{AO}{BO} \rightarrow AQ = 4.5 \times \frac{8}{6} = 6$ cm 1

C17. In $\triangle ABC$, P and Q are on AB and AC with $PQ \parallel BC$, $AP = 3$ cm and $PB = 6$ cm. Show that $\triangle APQ \sim \triangle ABC$ and find PQ if $BC = 12$ cm. [2]

$\angle A$ common, $\angle APQ = \angle ABC$ (corresponding): $\triangle APQ \sim \triangle ABC$ (AA) 1

$\frac{PQ}{BC} = \frac{AP}{AB} = \frac{3}{9} \rightarrow PQ = 4$ cm 1

C18. $\triangle ABC \sim \triangle PQR$, and AM, PN are their altitudes. Prove that $\frac{AM}{PN} = \frac{AB}{PQ}$. [2]

$\angle B = \angle Q$ and $\angle AMB = \angle PNQ = 90^\circ$: $\triangle ABM \sim \triangle PQN$ (AA) 1

So $\frac{AB}{PQ} = \frac{AM}{PN}$ 1

Theorem and proofs

C19. State and prove the Basic Proportionality Theorem. [5]

Statement: a line drawn parallel to one side of a triangle, meeting the other two sides in distinct points, divides them in the same ratio 1

Given $\triangle ABC$, $DE \parallel BC$; to prove $\frac{AD}{DB} = \frac{AE}{EC}$; construction: join BE, CD; draw $EN \perp AB$, $DM \perp AC$ (figure) 1

$\frac{\text{ar}(ADE)}{\text{ar}(BDE)} = \frac{AD}{DB}$ (same height EN) 1

$\frac{\text{ar}(ADE)}{\text{ar}(DEC)} = \frac{AE}{EC}$ (same height DM) 1

$\text{ar}(BDE) = \text{ar}(DEC)$ (same base DE, between the same parallels), so $\frac{AD}{DB} = \frac{AE}{EC}$ 1

C20. $ABCD$ is a trapezium with $AB \parallel DC$. E and F are points on AD and BC with $EF \parallel AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$. [3]

Join AC, meeting EF at G. In $\triangle ADC$, $EG \parallel DC$: $\frac{AE}{ED} = \frac{AG}{GC}$ 1

In $\triangle CAB$, $GF \parallel AB$: $\frac{CG}{GA} = \frac{CF}{FB}$, i.e. $\frac{AG}{GC} = \frac{BF}{FC}$ 1

Hence $\frac{AE}{ED} = \frac{BF}{FC}$ 1

C21. In $\triangle PQR$, $PS \perp QR$ with S between Q and R, and $PS^2 = QS \times SR$. Prove that $\angle QPR = 90^\circ$. [3]

$\frac{PS}{QS} = \frac{SR}{PS}$ and $\angle PSQ = \angle RSP = 90^\circ$: $\triangle QSP \sim \triangle PSR$ (SAS) 1

So $\angle QPS = \angle PRS$ and $\angle SPR = \angle PQS$; hence $\angle QPR = \angle PRQ + \angle PQR$ 1

Angle sum: $2\angle QPR = 180^\circ$, so $\angle QPR = 90^\circ$ 1

Applications: shadows and ladders

Q	Answer	Working / reason
C22	**(c)** 8 m	$\sqrt{17^2 - 15^2} = \sqrt{64} = 8$ m.

C23. A vertical stick 1.8 m long casts a shadow 1.2 m long. At the same time a tree casts a shadow 9 m long. Find the height of the tree. [2]

Similar triangles (AA): $\frac{h}{1.8} = \frac{9}{1.2}$ 1

$h = 13.5$ m 1

C24. A lamp is 6 m above the ground. A boy 1.5 m tall stands 9 m from the foot of the lamp post. Find the length of his shadow. [3]

Shadow s m; the triangles are similar (AA) 1

$\frac{1.5}{6} = \frac{s}{s+9} \rightarrow 4s = s + 9$ 1

$s = 3$ m 1

D

PART D · SOLUTIONS

Case-Based Questions – Solutions

Worked solutions with step marks for every question in Part D.

D1. Case Study · Shadow and Height [4]

(i) Why are the triangles formed by the student and the flagpole with their shadows similar? [1]

Both stand vertically (90°) and the sun's rays meet the ground at the same angle: AA 1

(ii) Find the height of the flagpole. [1]

$\frac{h}{1.6} = \frac{15}{2} \rightarrow h = 12$ m 1

(iii) Later in the day the flagpole's shadow is 9 m long. How long is the student's shadow then? [2]

Flagpole 12 m (from the shadows at 10 a.m.); $\frac{1.6}{12} = \frac{s}{9}$ 1

$s = 1.2$ m 1

OR

(iii) A tree 8 m high stands near the flagpole. How long is its shadow at 10 a.m.? [2]

Shadow = height $\times \frac{2}{1.6}$ 1

$8 \times 1.25 = 10$ m 1

D2. Case Study · Roof Truss [4]

(i) Find the length EC. **[1]**

$\frac{1.5}{2.5} = \frac{1.8}{EC} \rightarrow EC = 3$ m — 1

(ii) Find the length of the beam DE. **[1]**

$\frac{DE}{BC} = \frac{AD}{AB} = \frac{1.5}{4} \rightarrow DE = 2.25$ m — 1

(iii) A second beam $FG \parallel BC$ is fixed with F on AB and $AF = 3$ m. Find FG and AG. **[2]**

$AB = 4$, $AC = 4.8$ (from the given lengths). $\frac{FG}{6} = \frac{3}{4} \rightarrow FG = 4.5$ m — 1

$AG = 4.8 \times \frac{3}{4} = 3.6$ m — 1

OR

(iii) A builder joins a point 2 m from A on AB to a point 2.4 m from A on AC. Is this beam parallel to BC? Justify. **[2]**

$AB = 4$ m, $AC = 4.8$ m; $\frac{2}{4} = \frac{1}{2}$ and $\frac{2.4}{4.8} = \frac{1}{2}$ — 1

Equal ratios, so yes, it is parallel (converse of the BPT) — 1

D3. Case Study · Pinhole Camera [4]

(i) Which criterion shows $\triangle OAB \sim \triangle OA'B'$? **[1]**

AA: $\angle AOB = \angle A'OB'$ (vertically opposite) and $\angle B = \angle B' = 90^\circ$ — 1

(ii) Find the height of the object. **[1]**

$\frac{h}{3} = \frac{600}{10} \rightarrow h = 180$ cm $= 1.8$ m — 1

(iii) The object is moved to 9 m from the hole. Find the new height of the image. **[2]**

Object 180 cm (from the first set-up): $\frac{180}{x} = \frac{900}{10}$ — 1

$x = 2$ cm — 1

OR

(iii) The screen is moved to 15 cm behind the hole (object at 6 m). Find the height of the image. **[2]**

$\frac{x}{180} = \frac{15}{600}$ — 1

$x = 4.5$ cm — 1

D4. Case Study · Mirror Method [4]

(i) Why is $\triangle ABM \sim \triangle CDM$? **[1]**

$\angle B = \angle D = 90^\circ$ and $\angle AMB = \angle CMD$: AA — 1

(ii) Find the height of the tree. **[1]**

$\frac{h}{12} = \frac{1.5}{2} \rightarrow h = 9$ m — 1

(iii) Keeping 2 m from the mirror, how far from a pole 13.5 m high must the mirror be for Riya to see its top? **[2]**

$\frac{13.5}{d} = \frac{1.5}{2}$ — 1

$d = 18$ m — 1

OR

(iii) Her friend, whose eyes are 1.2 m above the ground, stands 1.6 m from the same mirror. Can she see the top of the tree? Justify. **[2]**

Tree 9 m (from (ii)): needs $\frac{1.2}{1.6} = \frac{9}{12}$ — 1

Both are 0.75, so yes, she can — 1

E

PART E · SOLUTIONS

Challenge Questions – Solutions

Worked solutions with step marks for every question in Part E.

E1. Through the midpoint M of side CD of parallelogram $ABCD$, the line BM is drawn, meeting AC at L and AD produced at E. Prove that $EL = 2BL$. **[3]**

$\triangle BMC \cong \triangle EMD$ (ASA: $CM = DM$, vertically opposite angles, alternate angles), so $DE = BC$ and $AE = 2BC$ — 1

$\triangle AEL \sim \triangle CBL$ (AA: alternate angles $\angle EAL = \angle BCL$, vertically opposite angles at L) — 1

$\frac{EL}{BL} = \frac{AE}{CB} = 2$, so $EL = 2BL$ — 1

E2. In $\triangle ABC$, AD is a median and E is the midpoint of AD. BE produced meets AC at F. Prove that $AF = \frac{1}{3}AC$. **[3]**

Draw $DG \parallel BF$ meeting AC at G. In $\triangle BCF$, D is the midpoint of BC, so G is the midpoint of CF: $FG = GC$ — 1

In $\triangle ADG$, E is the midpoint of AD and $EF \parallel DG$, so F is the midpoint of AG: $AF = FG$ — 1

$AF = FG = GC$, so $AF = \frac{1}{3}AC$ — 1

E3. In $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$. If $AD = 4$ cm and $CD = 9$ cm, find BD and AB. **[3]**

$\triangle ADB \sim \triangle BDC$, so $BD^2 = AD \cdot DC = 36 \rightarrow BD = 6$ cm — 1

$\triangle ADB \sim \triangle ABC$, so $AB^2 = AD \cdot AC = 4 \times 13$ — 1

$AB = \sqrt{52} = 2\sqrt{13}$ cm — 1

E4. Sides AB, BC and median AD of $\triangle ABC$ are proportional to sides PQ, QR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$. **[3]**

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, and $\frac{BC}{QR} = \frac{BD}{QM}$ (halves) — 1

So $\triangle ABD \sim \triangle PQM$ (SSS), giving $\angle B = \angle Q$ — 1

With $\frac{AB}{PQ} = \frac{BC}{QR}$: $\triangle ABC \sim \triangle PQR$ (SAS) — 1

E5. O is a point inside $\triangle PQR$. Points A, B, C on OP, OQ, OR are such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. **[3]**

In $\triangle OPQ$, $AB \parallel PQ$: $\dfrac{OA}{AP} = \dfrac{OB}{BQ}$ — 1

In $\triangle OPR$, $AC \parallel PR$: $\dfrac{OA}{AP} = \dfrac{OC}{CR}$ — 1

So $\dfrac{OB}{BQ} = \dfrac{OC}{CR}$; by the converse of the BPT in $\triangle OQR$, $BC \parallel QR$ — 1

E6. A girl 1.6 m tall stands 3.2 m from a lamp post and casts a shadow 4.8 m long. Find the height of the lamp above the ground. **[2]**

$\dfrac{1.6}{H} = \dfrac{4.8}{3.2 + 4.8}$ — 1

$H = \dfrac{1.6 \times 8}{4.8} = \dfrac{8}{3} \approx 2.67$ m — 1